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Error Analysis in Epicentre Determination for the Seismic Monitoring Network in Hong Kong

by

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Contents

Abstract

Error analysis was carried out to provide a general picture on the accuracy in epicentre determination of the seismic network in Hong Kong. Results showed that generally accuracy is improved as the number of seismic stations increases. It was also found that the arrival time of S-wave is crucial to improving the accuracy of determining the epicentre.

摘要

為了解香港地震監測網絡的震中定位準確度，進行了誤差分析。結果發現準確度隨地震站的增多而上升，地震 S 波的初抵時間對震中位置的準確度有顯著的影響。

Figures

1. Introduction

The Hong Kong Observatory installed a network of three short-period seismograph stations at Cheung Chau, Tsim Bei Tsui and Yuen Ng Fan in 1979 to monitor local or nearby earthquakes. In 1997, the network was upgraded to a digital system consisting of eight outstations. These stations are located at Cape D'Aguilar, Cheung Chau, Keung Shan, Lead Mine Pass, Luk Keng, Siu Lam, Tsim Bei Tsui and Yuen Ng Fan (Figure 1). Detailed description of the upgraded digital seismic system can be found in Lam (1998) and Tam (1997).

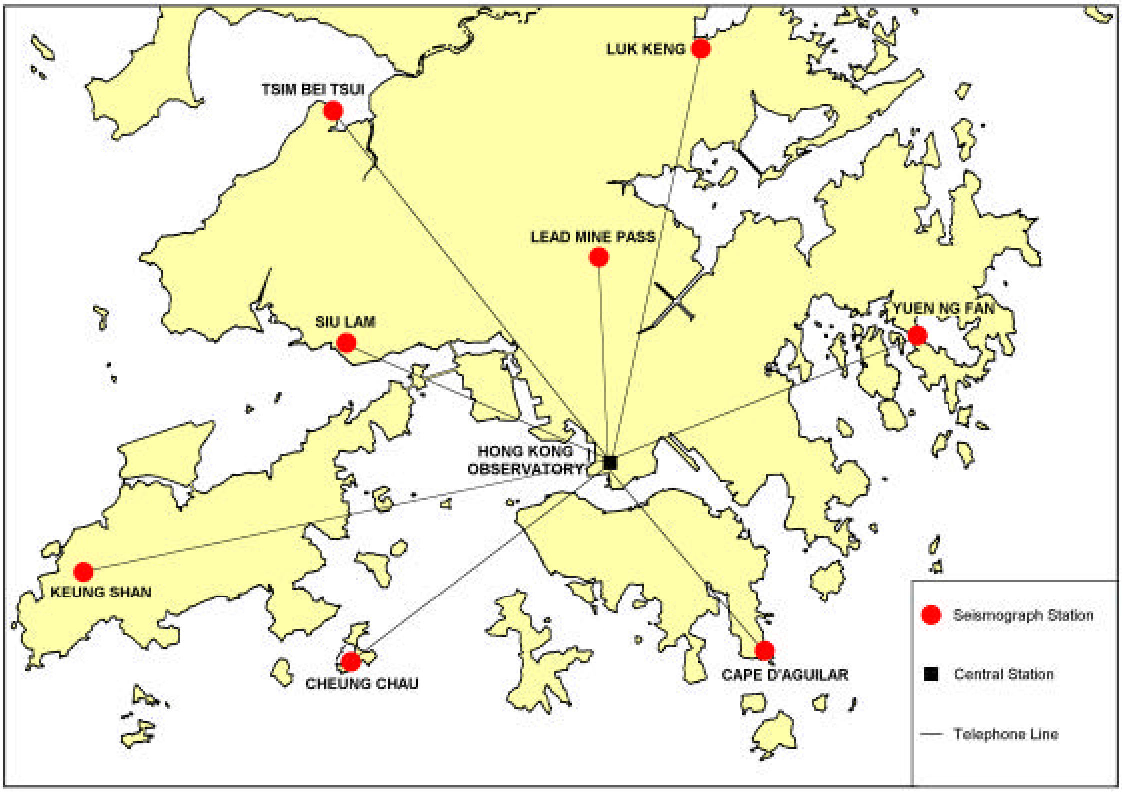

Figure 1. The Hong Kong seismic monitoring network

This short report serves to illustrate the improved accuracy attained by the upgraded network in epicentre determination by comparing its performance with the old seismic monitoring network. Error analyses on the old network was documented in Shun (1992).

The report is divided into four sections. This section serves to introduce the purpose of writing this report. Section 2 presents the methodology and the theory behind. Error analysis results are discussed in Section 3 and concluding remarks are given in the last section.

2. Methodology

The error analysis carried out in this note was similar to that given by Shun (1992). In order to make a direct comparison between the upgraded and old seismic network, the following assumptions as stated in Shun (1992) were made:

(a) Focal depths were taken to be zero
(b) The vertical z-coordinates of the stations were set to zero
(c) The crustal structure was simplified by a single horizontal layer of constant P and S velocity with V_p = 5.6 km/s and V_p/V_s = 1.78
(d) Accuracy of P-time was to the nearest 0.1 second
(e) Accuracy of S-time was to the nearest 0.1 second

For completeness, the method of analysis is stated below. For a small network such as the one in Hong Kong, the horizontal baseline is of the order of several tens of kilometers and curvature of the earth can be neglected. Hence Cartesian coordinate system can be used in the epicentre location problem.

Suppose X, Y, Z and T are respectively the coordinates of the hypocentre and the origin time of a given earthquake. The problem is to calculate these 4 parameters given a set of P or S arrival times t_k from stations at positions (x_k, y_k, z_k) where k = 1, 2, ..., n and n is the number of seismic stations.

For a given trial hypocentre (X*, Y*, Z*) and a trial origin time T*, a set of theoretical arrival times T_k can be obtained for a given crustal structure. From assumptions (a), (b) and (c), the theoretical arrival times are given by:

$$T_k = T^* + \sqrt{[(X^*- x_k)^2 + (Y^* - y_k)^2]} / V \quad \text{.........} (1)$$

where $V = V_p$ or V_s according to whether P or S arrival times are considered, $k = 1, 2, \ldots\ldots, n$.

Equation (1) can be rewritten as :

$$(X^* - x_k)^2 + (Y^* - y_k)^2 = V^2 (T_k - T^*)^2 \quad \ldots\ldots\ldots (2)$$

Suppose there are errors Γ_k in these P or S times and the resulting errors for X*, Y* and T* in solving equation (2) are respectively α, β and γ. Then

$$[(X^* + \alpha) - x_k]^2 + [(Y^* + \beta) - y_k]^2 = V^2 [(T_k + \Gamma_k) - (T^* + \gamma)]^2 \ldots (3)$$

Expanding equation (3), subtracting equation (2) and ignoring second order terms in α, β, γ and Γ_k, we get

$$(X^* - x_k)\alpha + (Y^* - y_k)\beta = (T_k - T^*)(\Gamma_k - \gamma)V^2 \quad \ldots\ldots\ldots (4)$$

Let $D_k = \sqrt{[(X^* - x_k)^2 + (Y^* - y_k)^2]} = V(T_k - T^*)$, we obtain

$$(X^* - x_k)\alpha + (Y^* - y_k)\beta + V D_k \gamma = V D_k \Gamma_k \quad \ldots\ldots\ldots (5)$$

Equation (5) is a system of n simultaneous linear equations in 3 unknowns α, β and γ. It can be written in matrix form

$$\mathbf{A}\,\mathbf{X} = \mathbf{B} \quad \ldots\ldots\ldots (6)$$

where $\mathbf{X} = (\alpha, \beta, \gamma)^T$, T denotes the transpose of the matrix.

When $n > 3$, we can multiply equation (6) by A^T and get

$$\mathbf{(A^T A)\,X} \quad = \quad \mathbf{A^T B} \qquad \text{........ (7)}$$

This is a system of 3 simultaneous linear equations in 3 unknowns. It can be solved by standard methods such as Gaussian Elimination.

The simpler case when only P-times are considered will be treated first and then the case in which both P- and S-times are taken into consideration.

Case (i) - only P times are considered

By assumption (d) mentioned before, we allow the errors Γ_k (k = 1, 2, ...n) to take on 3 different states :

$$\Gamma_k \;=\; -0.05,\ 0 \text{ or } 0.05 \text{ second}$$

For n stations, there is a total of 3^n combinations of the P-time errors. Set $V = V_p$ and solving equations (6) or (7) for these 3^n combinations, a set of results (α_i, β_i, γ_i) where i = 1, 2, 3, ..., 3^n can be obtained. The distant errors in the x and y directions α_i, β_i can be combined to yield a resultant distance error δ_i given by

$$\delta_i = \sqrt{(\alpha_i^{\,2} + \beta_i^{\,2})}\,, \qquad i = 1, 2, 3, \ldots\ldots, 3^n$$

The standard resultant distance error σ_δ is then given by

$$\sigma_\delta = \left(\Sigma_i\, \delta_i^{\,2}/3^n\right)^{1/2}, \qquad i = 1, 2, 3, \ldots\ldots, 3^n$$

Case (ii) - both P- and S-times are considered

The methodology is similar to case (i). The only change is to add the following m equations to the system of equations in (2):

$$(X^* - x_k)^2 + (Y^* - y_k)^2 = V_s^2 (T_k - T^*)^2$$

where V_s = 5.6/1.78 = 3.15 km/s, T_k (k = 1, 2, ….., m) are the S-times and m the number of stations for which S-times are used.

In this study, the standard errors were computed for epicentres at the vertices of a 2 km grid system with 10 201 grid points covering the area

$$-100 \text{ km} \leqq x, y \leqq 100 \text{ km}$$

with the Hong Kong Observatory's Headquarters at the origin. The x and y coordinates for the stations were computed from their latitudes and longitudes by a method described in Richter (1958).

3. Results

Computer programs were written to carry out the error analysis as mentioned in section 2.

For simplicity, the following abbreviations for the eight seismic stations are used.

Station	Abbreviation
Cape D'Aguilar	CD
Cheung Chau	CC
Keung Shan	KS
Lead Mine Pass	LMP
Luk Keng	LK
Siu Lam	SL
Tsim Bei Tsui	TBT
Yuen Ng Fan	YNF

Error analyses for the cases with three, four, six and eight P-times were conducted and plots of standard resultant distance error σ_{δ} are shown in Figures 2 to 6 respectively.

Figure 2 corresponds to the error field for the old network. Although the standard distance errors are relatively small within the triangle formed by these three stations, there are 6 distinctive narrow strips lying along the external extensions of the triangle for which the errors can be quite large (more than 100 km). A detailed mathematical explanation of this can be found in Shun (1992).

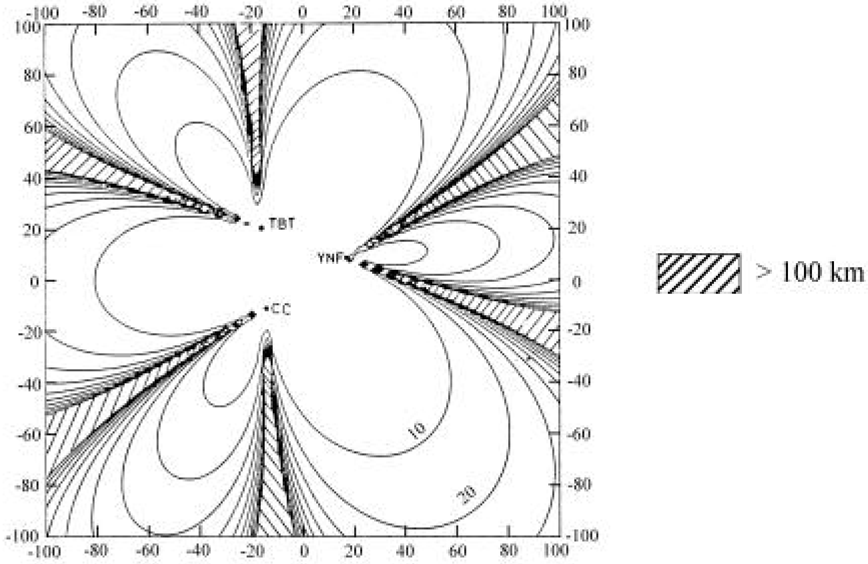

Figure 2. Standard distance error field for a network of 3 stations (in km)
(P-time accuracy 0.1s for CC, TBT and YNF)

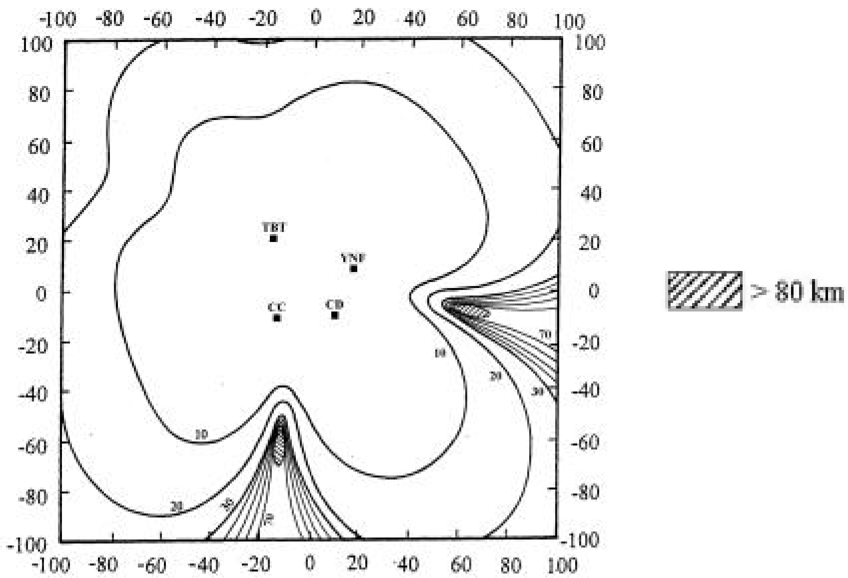

Figure 3. Standard distance error field for a network of 4 stations (in km)
(P-time accuracy 0.1s for CC, TBT, YNF and CD)

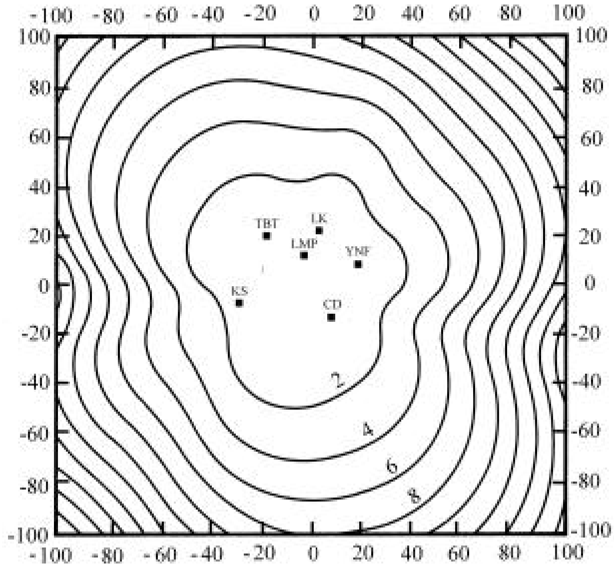

Figure 4. Standard distance error field for a network of 6 stations (in km)
(P-time accuracy 0.1s for TBT, YNF, CD, KS, LMP and LK)

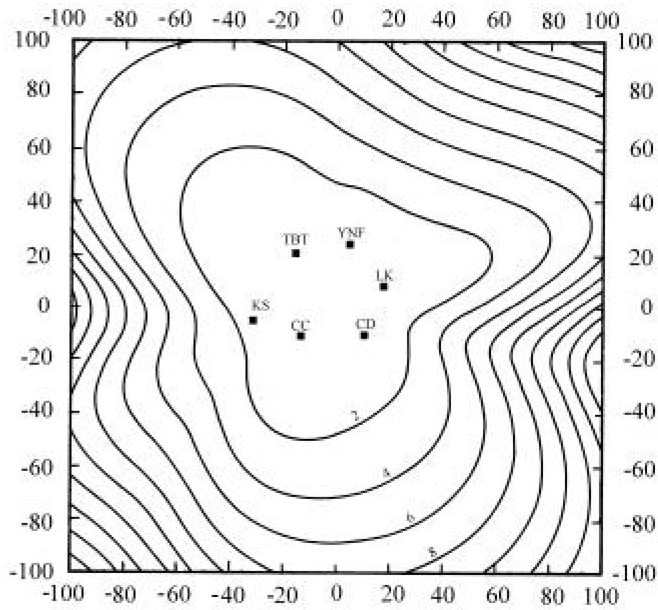

Figure 5. Standard distance error field for a network of 6 stations (in km)
(P-time accuracy 0.1s for CC, TBT, YNF, CD, KS and LK)

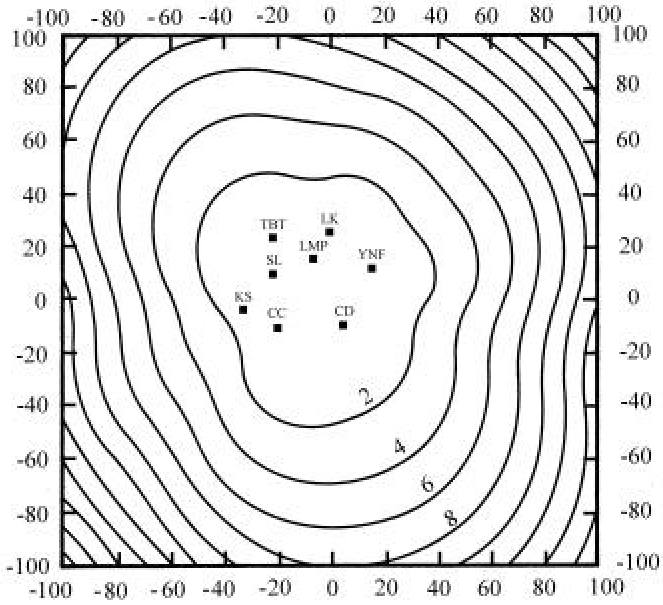

Figure 6. Standard distance error field for a network of 8 stations (in km)
(P-time accuracy 0.1s for all stations)

From Figures 2 to 6, it can be clearly seen that the standard distance error for an epicentre lies within the network of stations is smaller than that lying outside. The standard error at a grid point generally decreases as the number of station increases. For epicentres outside the network, the improvement in accuracy is generally more significant from 3 to 6 stations than from 6 to 8 stations. There are still areas of extremely large error found for the 4 stations but the areas disappear in cases of 6 and 8 stations. It is interesting to note that areas of large error for 4 stations occur around the intersection points of extrapolated opposite sides of the quadrilateral with the 4 stations as vertices (Figure 3). A detailed mathematical proof can be found in the Appendix. Comparing Figures 4 and 5, the distribution of standard errors is rather similar and the errors mainly depend on the distance of the grid points from the origin.

Comparing Figures 2 and 6, it is found that the standard errors for the upgraded network are much less than that of the old network. Even if two stations malfunctioned or P-times could not be determined, the accuracy is still much better than the old network (compare Figures 2 and 4).

The above discussion is limited to cases when only P-times are recorded. Figures 7 to 11 show the standard distance error fields when both P and S-times are present.

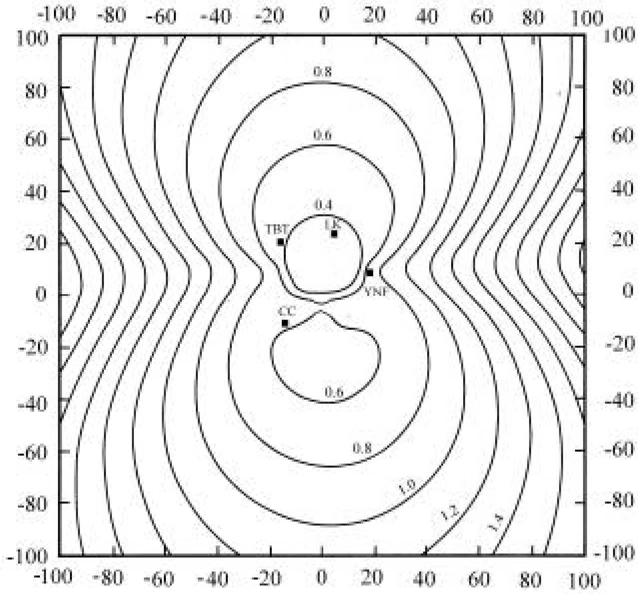

Figure 7. Standard distance error field for a network of 4 stations (in km)
(P-time accuracy 0.1s for CC, TBT, YNF, LK, S-time accuracy 0.1s for TBT)

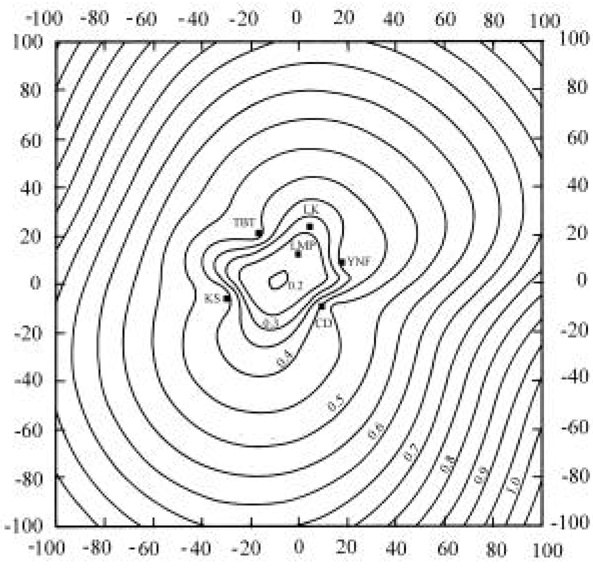

Figure 8. Standard distance error field for a network of 6 stations (in km)
(P-time accuracy 0.1s for TBT, YNF, CD, KS, LMP, LK, S-time accuracy 0.1s for TBT)

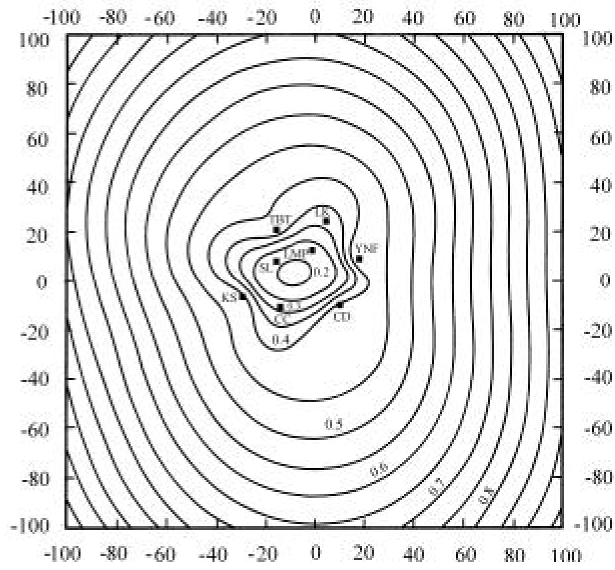

Figure 9. Standard distance error field for a network of 8 stations (in km)
(P-time accuracy 0.1s for all stations, S-time accuracy 0.1s for TBT)

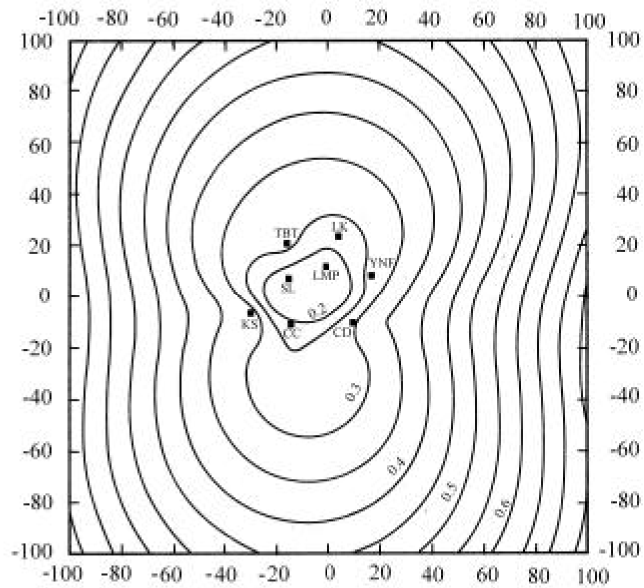

Figure 10. Standard distance error field for a network of 8 stations (in km)
(P-time accuracy 0.1s for all stations, S-time accuracy 0.1s for TBT, YNF, LMP, LK)

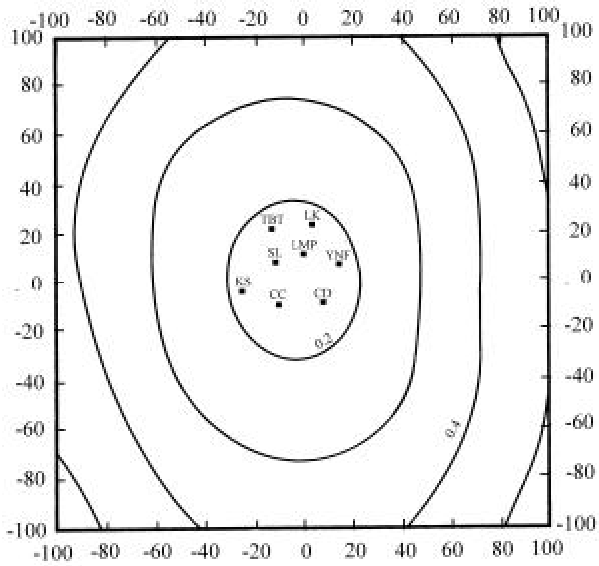

Figure 11. Standard distance error field for a network of 8 stations (in km)
(P-time accuracy 0.1s for all stations, S-time accuracy 0.1s for all stations)

Comparing Figures 3 and 7, 4 and 8, and also 6 and 9, it can be seen that the addition of one S-time can greatly reduce the standard error. The accuracy in Figure 7 is even better than Figure 6 indicating that the introduction of one S-time may be more significant than the addition of 4 P-times. With the increase in the number of S-time used as in Figure 10, the accuracy is enhanced. To save computer resources, a 50 km grid system instead of 2 km grid system was adopted to run the case of 8 P- and 8 S-times. Figure 11 is a rough sketch of the results. In this ideal case in which 8 P- and 8 S-times can be utilized, the largest standard error within the grid area is only 0.56 km, attained at the point (100,100).

4. Conclusion and Discussion

This report serves to provide a guideline on the accuracy in epicentre determination of the upgraded seismic system as related to recorded P- and S-times.

From the results shown in Section 3, the accuracy of epicentre determination generally improves as the number of recording station increases. S-times are more crucial in the accurate determination of location than that contributed by P-times. Hence, the standard location error of the new 8-station seismic system is less than that of the old 3-station network.

It should be noted that the results given in Section 3 depend on a number of assumptions. The assumptions are, single-layered structure of the earth crust with uniform P- and S-velocities, the depth of hypocentre and the recording stations are all taken to be zero and also the accuracy in P- and S-times are all to be within 0.1s.

In a real situation, the structure of the earth crust is far more complicated and it is not realistic to assume the depth of the hypocentre to be zero, especially for ' deep' earthquakes. The accuracy of P- and S-times will vary from case to case. For a larger earthquake, P- and S-times will be easier to determine. For a small earthquake, it is more difficult to identify the P- and S-phases at stations with relatively large background noise and hence larger errors may result. The identification of S-time would require an experienced analyst and the errors often found to be larger than that of P-time.

Acknowledgement

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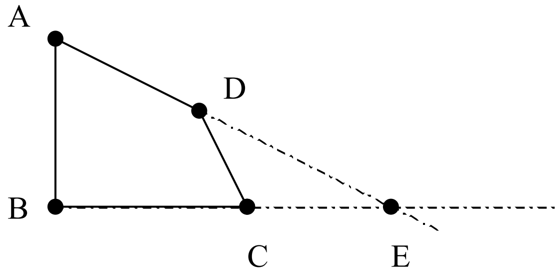
Appendix

A mathematical proof for extremely large errors found for 4 stations is shown below.

Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ be the coordinates of the 4 seismic stations and $E(X, Y)$ be the intersection point of the extrapolated opposite sides AD and BC as shown below,



then
$$\frac{Y-y_1}{X-x_1} = \frac{Y-y_4}{X-x_4} = m_1 \text{..........(8)}$$
$$\frac{Y-y_2}{X-x_2} = \frac{Y-y_3}{X-x_3} = m_2 \text{..........(9)}$$

where m_1 , m_2 are the slopes of AD and BC respectively.

Matrix A of equation (6) in Section 2 can be written as:

$$A = \begin{pmatrix} X-x_1 & Y-y_1 & V\sqrt{(X-x_1)^2+(Y-y_1)^2} \\ X-x_2 & Y-y_2 & V\sqrt{(X-x_2)^2+(Y-y_2)^2} \\ X-x_3 & Y-y_3 & V\sqrt{(X-x_3)^2+(Y-y_3)^2} \\ X-x_4 & Y-y_4 & V\sqrt{(X-x_4)^2+(Y-y_4)^2} \end{pmatrix}$$

By substituting (8) and (9),

$$A = \begin{pmatrix} X-x_1 & m_1(X-x_1) & V\sqrt{1+m_1^2}\ \left|X-x_1\right| \\ X-x_2 & m_2(X-x_2) & V\sqrt{1+m_2^2}\ \left|X-x_2\right| \\ X-x_3 & m_2(X-x_3) & V\sqrt{1+m_2^2}\ \left|X-x_3\right| \\ X-x_4 & m_1(X-x_4) & V\sqrt{1+m_1^2}\ \left|X-x_4\right| \end{pmatrix}$$

Since E is an external intersection point, $(X-x_1)(X-x_4)>0$ and $(X-x_2)(X-x_3)>0$. There are four possible cases: (i) $X-x_1>0$, $X-x_4>0$, $X-x_2>0$, $X-x_3>0$. (ii) $X-x_1>0$, $X-x_4>0$, $X-x_2<0$, $X-x_3<0$. (iii) $X-x_1<0$, $X-x_4<0$, $X-x_2>0$, $X-x_3>0$. (iv) $X-x_1<0$, $X-x_4<0$, $X-x_2<0$, $X-x_3<0$.

Consider case (i)

$$A=\begin{pmatrix} X-x_1 & m_1(X-x_1) & V\sqrt{1+m_1^2}\ (X-x_1) \\ X-x_2 & m_2(X-x_2) & V\sqrt{1+m_2^2}\ (X-x_2) \\ X-x_3 & m_2(X-x_3) & V\sqrt{1+m_2^2}\ (X-x_3) \\ X-x_4 & m_1(X-x_4) & V\sqrt{1+m_1^2}\ (X-x_4) \end{pmatrix}$$

$$A^T A=\begin{pmatrix} D_1^2+D_2^2 & m_1D_1^2+m_2D_2^2 & V\sqrt{1+m_1^2}D_1^2+V\sqrt{1+m_2^2}D_2^2 \\ m_1D_1^2+m_2D_2^2 & m_1^2D_1^2+m_2^2D_2^2 & Vm_1\sqrt{1+m_1^2}D_1^2+Vm_2\sqrt{1+m_2^2}D_2^2 \\ V\sqrt{1+m_1^2}D_1^2+V\sqrt{1+m_2^2}D_2^2 & Vm_1\sqrt{1+m_1^2}D_1^2+Vm_2\sqrt{1+m_2^2}D_2^2 & V^2(1+m_1^2)D_1^2+V^2(1+m_2^2)D_2^2 \end{pmatrix}$$

where $D_1^2=(X-x_1)^2+(X-x_4)^2$, $D_2^2=(X-x_2)^2+(X-x_3)^2$.

Denote the determinant of $A^T A$ by $\left|A^T A\right|$, by using row operations $m_1R_1-R_2$ and $V\sqrt{1+m_1^2}R_1-R_3$,

$$\left|A^T A\right|=\begin{vmatrix} D_1^2+D_2^2 & m_1D_1^2+m_2D_2^2 & V\sqrt{1+m_1^2}D_1^2+V\sqrt{1+m_2^2}D_2^2 \\ (m_1-m_2)D_2^2 & m_2(m_1-m_2)D_2^2 & V(m_1-m_2)\sqrt{1+m_2^2}D_2^2 \\ V\left(\sqrt{1+m_1^2}-\sqrt{1+m_2^2}\right)D_2^2 & Vm_2\left(\sqrt{1+m_1^2}-\sqrt{1+m_2^2}\right)D_2^2 & V^2\sqrt{1+m_2^2}\left(\sqrt{1+m_1^2}-\sqrt{1+m_2^2}\right)D_2^2 \end{vmatrix}$$

$$=V(m_1-m_2)\left(\sqrt{1+m_1^2}-\sqrt{1+m_2^2}\right)D_2^4\begin{vmatrix} D_1^2+D_2^2 & m_1D_1^2+m_2D_2^2 & V\sqrt{1+m_1^2}D_1^2+V\sqrt{1+m_2^2}D_2^2 \\ 1 & m_2 & V\sqrt{1+m_2^2} \\ 1 & m_2 & V\sqrt{1+m_2^2} \end{vmatrix}$$

$= 0$ (since Row 2 and Row 3 are equal)

By similar computation, $\left|A^T A\right|=0$ for cases (ii), (iii) and (iv).

Hence from equation (7) in Section 2 and by Cramer's rule, the error would be infinity at the point $E(X,Y)$.