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NUMERICAL SIMULATION OF SHALLOW WATER TIDES IN HONG KONG

by

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1. INTRODUCTION

Tidal prediction information is required by private sectors for various purposes. With water level data being collected by the Royal Observatory at nine permanent tide stations in Hong Kong (Figure 1), predictions at these locations may be made through harmonic analysis (HA) of the measured data. As different sectors of the community may require information for different locations, predictions should preferably be made at as many locations as possible. While the installation of additional permanent tide gauges will provide measured data for more locations for analysis and hence prediction, it is uneconomical to do so. On the other hand, temporary gauges will not provide sufficient quantity of data for analysing inter-annual variations due to meteorological effects which are significant in Hong Kong. Without measured data, tides can be predicted either by the tide tables (TT) approach (Royal Observatory 1988, Hydrographer of the Navy 1989) or by using numerical models (Flather and Heaps 1975).

In the TT approach, measured tide levels at a reference station are first analysed by the HA method. The harmonic constituents resolved are then used to predict daily high/low waters for the reference station which in turn are adjusted in time and height to give levels for the location of interest. If limited data is available at the location concerned, the time/height adjustments (THA) to be applied to different water levels will be determined from correlations with levels at the reference station. Otherwise, the THA have to be estimated from information at nearby stations, if any. In the former case, the accuracy of the prediction depends very much on the sample size of data used in the correlation. In the latter case, resemblance of tidal characteristics of the location concerned with the nearby station is vital. In Hong Kong, Quarry Bay (QUB) is taken as the reference station and the TT approach is likely to work well at locations where semi-diurnal tides are, similar to QUB, observed most of the time. For places like Tai Po Kau (TPK) where shallow water characteristics in the form of double high waters (Pugh 1987) are exhibited, the TT approach is likely to be less reliable (Poon and Chiu 1988).

Tam et al (1989) carried out a feasibility study on the use of a vertically integrated hydrodynamical model to simulate tides at TPK. The results were encouraging but also pointed to the need to :

(a) calibrate the smoothing parameter and the coefficient of bottom friction as used in the model; and
(b) use a method, different from that adopted in previous numerical modelling studies, to generate input water levels for the open boundary of the model.

With the ultimate goal of running the numerical model for predicting tides at places without measured data, experiments are carried out in this study following recommendations of Tam et al

(1989). To facilitate more reliable verification of the model computations, simulations are carried out again for TPK because (a) tidal characteristics there are significantly different from those at QUB; (b) it is located in a bay with only one opening to the ocean, thus rendering tuning of the model easier to control; and (c) both the HA and TT approaches have been applied to predict tides at TPK (Poon and Chiu 1988) and more meaningful comparisons of model simulations can therefore be made.

As the main purpose of the study is to simulate astronomical tides, meteorological effects are ignored by setting the wind to calm in all experiments.

2. DATA

(A) Coastline and Bathymetry

Coastline and bathymetric data are extracted from maps published by the Hong Kong Government and from Admiralty Chart No. 3026 (Hydrographer of the Navy 1966).

(B) Tide Levels

Harmonic constituents have been analysed from measured tide levels at QUB for the period from 1968 to 1986 and for TPK for the year 1973 using the HA method. Predictions with these harmonic constituents are taken as the "ground truth" (hereafter referred to as the expected levels) for evaluating the accuracy of model simulations and for deriving input data for the open boundary of the model following procedures described in section 3(C).

3. MODEL FORMULATION

(A) The Hydrodynamical Equations

The numerical model used is a vertically integrated hydrodynamical model adapted by Lau (1980) for simulating storm surges in Hong Kong. The equations are :

$$\frac{\partial U}{\partial t} - fV - \nu\nabla^2 U + \frac{r}{H} U \sqrt{U^2 + V^2} + g \frac{\partial \zeta}{\partial x} = \rho_w \frac{\tau^x}{H} \qquad (1)$$

$$\frac{\partial V}{\partial t} + fU - \nu\nabla^2 V + \frac{r}{H} V \sqrt{U^2 + V^2} + g \frac{\partial \zeta}{\partial y} = \rho_w \frac{\tau^y}{H} \qquad (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (HU) + \frac{\partial}{\partial y} (HV) = 0 \qquad (3)$$

where x, y = space co-ordinates

t = time

U, V = components of tidal current

ζ = water surface elevation

H = total depth

τ^x, τ^y = components of wind stress

g = acceleration due to gravity

f = Coriolis parameter

r = coefficient of bottom friction

ν = coefficient of horizontal eddy viscosity

$\tau^x = \rho_a \lambda W_x \sqrt{W_x^2 + W_y^2}$

$\tau^y = \rho_a \lambda W_y \sqrt{W_x^2 + W_y^2}$

ρ_a = air density

ρ_w = water density

λ = wind drag coefficient

W_x, W_y = components of wind speed

The flow is assumed to be incompressible and well mixed. Vertical accelerations are considered negligible. Spatial variations in the atmospheric pressure and Coriolis parameter are neglected as the areas under study are small. The finite difference scheme and staggered grid system employed to solve the equations are detailed in Lau (1980).

At the end of every time step, the water level and current at a grid point are smoothed by taking a five-point weighted average (Lau 1980). The weight given to the grid point concerned is called the smoothing parameter α. A smaller α will give a smoother field.

(B) Initial Condition

Water elevations and currents are set to zero initially. A lead up time of about twelve hours is usually required for water elevations and currents to attain equilibrium. For this reason, simulation results for the first twelve hours will be discarded when computing statistics of the residuals, i.e. differences between the simulated and expected levels.

(C) Boundary Treatment

The land boundary is assumed to be perfectly reflecting and the normal component of the current is always set to zero. At the open boundary, the normal component of the current is computed from water levels while the tangential component of the current is taken to be zero.

In all previous research work with the model at the Observatory, a constant THA was applied to the hourly tide levels of a reference station to obtain input for the open boundary. In this study, the TT approach is adopted with THA varying with water levels. For each boundary point, a pair of adjustment values (one for the height and one for the time) are estimated for an arbitrary higher high water and another pair for a lower low water. Adjustments for other levels are obtained by linear interpolation. With these adjustments, hourly water levels of the reference station are converted to levels at the boundary. Levels at any time step are computed by linear interpolation. Since the adjustments control the boundary input and therefore model output, they can be varied to minimize simulation errors and the initial choice is hence unimportant. The effect of this new approach will be discussed in section 4(A)(a)(i).

The choice of the reference station depends on the type of tide considered to be dominating at the boundary. If semi-diurnal tides are expected, QUB should be used. If TPK is chosen, multi-diurnal tides with double high waters will be input. As measured tides are not available at the boundary for determining the type of tide there, QUB and TPK will be used as the reference station in different experiments and the results discussed.

(D) The Smoothing Parameter and Coefficient of Bottom Friction

Output of the model may be tuned by varying the smoothing parameter α and coefficient of bottom friction r. α is varied from 0.9 to 0.998 in different tests and r is allowed to vary from 0.001 to 0.027.

4. THE TIDAL SIMULATION EXPERIMENTS

(A) Model TOLO4

A model with 24x36 rectangular grid points covering Tolo Harbour at a grid length of 0.42 km is first used (Figure 2). The basin is oriented in such a way that the x-axis is parallel to the Tolo Channel to enable smoother flow of water into and out of the harbour (Oceanography Department, Environmental Prediction Research Facility 1974). For ease of reference, this model will hereafter be referred to as the TOLO4, with "TOLO" signifying Tolo Harbour and "4" a grid length of about 0.4 km.

Details of experiments carried out with TOLO4 are described below. With a time step of 9 s, the computer time required to simulate a continuous four-day cycle on the Data General MV20000 of the Observatory was about 30 CPU minutes.

Poon and Chiu (1988) employed both the HA and TT methods to predict tides at TPK) for the year 1987 and found the root-mean-square error (RMSE) to be 0.175 m and 0.352 m respectively. As the modelling approach cannot be expected to outperform the HA method, model simulations with a RMSE of less than about 0.2 m are considered superior to the TT approach and therefore acceptable for prediction purposes.

(a) TPK as the reference station for the boundary

(i) Boundary values varying with water levels

To understand the effect of a constant THA, a spring tide event during the period from 27 to 30 March 1983 was simulated using $r = 0.003$ and $\alpha = 0.98$ (Run A1). Simulated fifteen-minute tide levels for the last two days of the event as well as the expected levels are plotted in Figure 3. A statistical summary of the residuals is given in Table 1.

Figure 4 shows the results of Run A2 which uses a different but still constant THA. Although a displacement was observed in the simulated curve in both the time and height axes compared with Figure 3, the time and height differences between the high and low waters remained constant.

To overcome the deficiency of the constant THA approach in simulating the varying tidal period and range, the TT approach described in section 3(C) is adoped in Run A3. Variations in the tidal period and range are now observed in Figure 5.

(ii) Choice of α and r

Employing the same α and boundary values as Run A3, Run A4 and Run A5 were carried out with $r = 0.01$ and 0.015 respectively. Figures 5 to 7 indicate that the double high waters become more pronounced for a smaller r.

Figures 8 to 10 show results of Run A6 to Run A8 with $r = 0.027$ and $\alpha = 0.9$, 0.98 and 0.998 respectively. Boundary values have been tuned for more accurate results. The double peaks were found to be amplified for a larger α. From Table 1, $\alpha = 0.998$ and $r = 0.027$ as used in Run A8 appear to be a reasonable choice for the spring tide case, with the RMSE of 0.075 m being much smaller than the target value of 0.2 m.

The same set of input for Run A8 applied in a neap tide case from 23 to 26 August 1986 (Run A9, Figure 11) produces a small RMSE of 0.045 m too. This renders support to the use of the above-mentioned α and r values.

It should be noted that the α and r chosen are by no means the best combination. The use of other values could produce results of comparable or even higher accuracies.

(b) QUB as the reference station for the boundary

With QUB as the reference station, several tests were carried out with different α and r. Again boundary adjustments have been changed to achieve more accurate simulations. Figure 12 plots the results of Run A10 for the 1983 case with $\alpha = 0.98$ and $r = 0.001$. The RMSE was 0.090 m.

On running A9 for the 1986 case, simulated levels (results not shown) were lower than the expected by about 0.1 to 0.2 m. Since the observed monthly mean sea levels at TPK for March 1983 and August 1986 are 1.22 m and 1.49 m respectively while those at QUB are 1.33 m and 1.46 m, adoption of boundary values based on QUB levels for March 1983 should produce an error of 0.14 m (=(1.49-1.22) - (1.46-1.33)) for August 1986. As differences in monthly mean sea levels between stations are not accounted for in the boundary adjustment process, the model cannot be expected to simulate the variations successfully. This means that if the reference station (QUB in this case) is far away from the boundary and sea levels are expected to be significantly different, the difference has to be accounted for by adding a constant offset or by tuning the boundary until the simulated and expected levels match.

Applying a constant offset of 0.2 m in the boundary values, Run A11 for the 1986 case produced a RMSE of 0.095 m (Figure 13).

(c) Choice between TPK and QUB as the reference station

The use of TPK has produced slightly smaller errors. Besides, its use would make the adjustment for differences in sea levels mentioned in the last sub-section unnecessary. However considering that the model would eventually be applied to other areas for which HA predictions are absent , the standard port QUB seems to be a more reasonable choice.

(B) Model TOLO8

In order to assess the effect of resolution on simulation errors, a coarser model with 13x18 grid points (Figure 14) at a grid length of 0.85 km was constructed to cover the same area as TOLO4. The time step used is 18 s. Based on a nomenclature similar to that mentioned in the last section, this model will be referred to as the TOLO8. CPU time required to run the model is about 3 minutes. The following model runs were made :

(a) Run B1 (Figure 15) -- TPK as reference, $\alpha = 0.998$, $r = 0.027$, 1983 case, boundary same as Run A8;
(b) Run B2 (Figure 16) -- TPK as reference, $\alpha = 0.998$, $r = 0.027$, 1986 case, boundary same as Run A9;
(c) Run B3 (Figure 17) -- QUB as reference, $\alpha = 0.98$, $r = 0.001$, 1983 case, boundary same as Run A10; and
(d) Run B4 (Figure 18) -- QUB as reference, $\alpha = 0.98$, $r = 0.001$, 1986 case, boundary same as Run A11.

As indicated in Table 1, the maximum errors of simulation with TPK as the reference station are larger than respective runs of TOLO4 but the RMSE are smaller. When QUB is used as the reference station, the errors are larger.

A TOLO2 was also tested (results not shown) and the accuracies of the simulations are comparable to those of TOLO4. Taking into consideration both accuracy and computer time, 0.4 or even 0.8 km may be the optimum grid length that should be used for modelling Tolo Harbour.

(C) Model MIRS4

The model area of TOLO4 was extended to cover the entire Mirs Bay with the same grid length of 0.42 km and time step of 9 s (MIRS4). The grid net is shown in Figure 19. QUB is used as the reference station in this model because tidal characteristics at the boundary are likely to be quite different from TPK. A CPU time of about 130 minutes are required to run this model. Using $\alpha = 0.98$ and $r = 0.001$, results of the two runs made are :

(a) Run C1 for the 1983 case -- despite the use of the same resolution as TOLO4, the double high waters at TPK are not as well simulated (Figure 20) as in the case of TOLO4 (Figure 12), but are better depicted than TOLO8 (Figure 17).

(b) Run C2 for the 1986 case -- results for TPK in Figure 21 are more accurate than those in Figure 18 but they do not compare well with those in Figure 13.

One reason for the less satisfactory results of MIRS4 compared with TOLO4 may be that the flow of water into and out of the narrow mouth of Tolo Harbour is more difficult for a model covering a larger area to simulate.

Simulations with MIRS2 (results not shown) indicate insignificant improvement over MIRS4, suggesting that the optimum grid length for this area is about 0.4 km.

5. HARMONIC ANALYSIS OF MODEL SIMULATED TIDE LEVELS

In order to further assess whether the model has skill in reproducing shallow water tides at TPK with QUB serving as the reference station, tides for the entire arbitrarily chosen month of August 1988 were simulated or predicted using TOLO4, α = 0.98 and r = 0.001. On analysing the results by the HA approach, a total of 41 harmonic constituents were resolved. The amplitudes of some shallow water constituents (Schureman 1958, Pugh 1987) are listed in Table 2.

For comparison purposes, the following data sets were also analysed and the harmonic constituents tabulated in Table 2 :

(a) water levels for TPK for August 1988 predicted by the TT approach;
(b) measured levels for TPK for August 1988; and
(c) measured levels for QUB for August 1988.

Harmonic constituents extracted from 1973 TPK measured levels and from 1968 to 1986 measured levels for QUB, as used in preparing the Tide Tables for Hong Kong are reproduced in Table 2 as well.

Similar to the observations of Poon and Chiu (1988), amplitudes of shallow water constituents for TPK are on average 2 to 3 times higher than those for QUB. Model simulated levels also indicate much more significant shallow water effects than those generated by the TT approach.

Table 3 also indicates the more significant role of shallow water effects at TPK in that M6/M4 and M4/M2 are much larger for TPK than for QUB. The magnitudes of the ratios computed for the model simulations are also much larger than those for the TT predicted levels.

The different sets of tide levels predicted for TPK for August 1988 were compared with each other and the differences analysed. From the results in Table 4, model simulated levels were found to differ least from HA predicted levels.

The above analyses show that the model is producing more accurate and meaningful predictions for TPK than the TT approach.

6. CONCLUDING REMARKS

Through numerous test runs carried out with TOLO4, values of the smoothing parameter and the coefficient of bottom friction that work well in both a spring and a neap tide scenario were obtained. If QUB is taken as the reference station for the boundary levels, then $\alpha = 0.98$, $r = 0.001$ are recommended. With TPK tides input at the boundary, $\alpha = 0.998$ and $r = 0.027$ are preferred. While more accurate simulations were produced with TPK as the reference, there is merit in using QUB at the boundary for application to other areas for which HA are not carried out.

The adoption of variable time/height adjustments for the boundary is shown to have led to more realistic and accurate simulations.

Statistically speaking, the water levels at TPK are reasonably well predicted with RMSE smaller than 0.2 km in all cases. Harmonic analysis of model simulations also indicated that shallow water effects absent in TT predicted levels can be reproduced using the model.

Based on the experiment results, it may be concluded that a grid length of 0.4 km is about the optimum that should be adopted for this numerical model. For a model covering a smaller area such as the Tolo Harbour, 0.8 km may be used.

As expected, TOLO4 performed better than MIRS4 because of the smaller area covered by the model. One model covering the entire Hong Kong is unlikely to give results of a very high accuracy. Hence if tidal prediction should be made with this model, it should be carried out area by area. In order to account for seasonal variations in the sea level, the model area should be chosen such that measured data is available inside the area.

Having produced reliable predictions for TPK, the model is expected to be able to predict tides at any location within the model area with reasonable accuracy too. Predictions of tides for one complete year take less than five hours of CPU time with TOLO8. This is considered more cost-effective than the TT approach which requires a known relationship between the tide levels at a location and those at QUB and the HA approach which needs a long series of tide data.

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Table 1 Statistical summary of errors of model simulation of TPK tides

Run No.	Year	Smoothing Parameter	Bottom Friction	Boundary Reference Station	Model	Errors (m)		
						Maximum	Mean	RMS
A1	1983	0.98	0.003	TPK	TOLO4	0.46	0.159	0.248
A2	1983	0.98	0.003	TPK	TOLO4	-0.44	-0.097	0.211
A3	1983	0.98	0.003	TPK	TOLO4	0.25	0.016	0.124
A4	1983	0.98	0.01	TPK	TOLO4	0.23	0.015	0.084
A5	1983	0.98	0.015	TPK	TOLO4	0.22	0.013	0.099
A6	1983	0.9	0.027	TPK	TOLO4	0.25	-0.005	0.098
A7	1983	0.98	0.027	TPK	TOLO4	0.22	-0.003	0.08
A8	1983	0.998	0.027	TPK	TOLO4	0.21	-0.001	0.075
A9	1986	0.998	0.027	TPK	TOLO4	0.1	-0.014	0.045
A10	1983	0.98	0.001	QUB	TOLO4	-0.18	0.021	0.09
A11	1986	0.98	0.001	QUB	TOLO4	0.22	-0.016	0.095
B1	1983	0.998	0.027	TPK	TOLO8	-0.24	-0.002	0.01
B2	1986	0.998	0.027	TPK	TOLO8	0.15	0.01	0.041
B3	1983	0.98	0.001	QUB	TOLO8	0.4	0.024	0.149
B4	1986	0.98	0.001	QUB	TOLO8	0.39	-0.016	0.167
C1	1983	0.98	0.001	QUB	MIRS4	-0.3	-0.05	0.134
C2	1986	0.98	0.001	QUB	MIRS4	-0.33	-0.048	0.142

Table 2 Amplitudes of some shallow water constituents for TPK and QUB

Shallow water constituent	TPK				QUB	
	1973 measured tides	Measured levels for August 1988	TT predicted levels for August 1988	Model simulated levels for August 1988	19-year measured levels	Measured levels for August 1988
MSF	0.0035	0.0647	0.013	0.0244	0.0107	0.0308
MO3	0.0181	0.0251	0.005	0.0103	0.0082	0.0175
MK3	0.0207	0.0229	0.0138	0.0181	0.0133	0.0133
SK3	0.0143	0.0085	0.001	0.0086	0.0065	0.0046
MN4	0.0269	0.037	0.0113	0.0206	0.0126	0.0159
M4	0.0799	0.135	0.0198	0.0524	0.0345	0.0552
MS4	0.0475	0.0893	0.0132	0.0345	0.0206	0.0391
S4	0.0016	0.0054	0.0024	0.0022	0.0021	0.0034
2MN6	0.0211	0.0242	0.0011	0.0117	0.002	0.0029
M6	0.037	0.0382	0.0015	0.0189	0.0035	0.0056
2MS6	0.0357	0.0345	0.0012	0.0199	0.0034	0.0058
2SM6	0.0093	0.0103	0.0006	0.0131	0.0025	0.0032
M8	0.002	0.0023	0.0006	0.0017	0.0012	0.0015
3MS8	0.0012	0.0047	0.001	0.003	0.0018	0.0021
MEAN	0.0228	0.0359	0.0061	0.0171	0.0088	0.0144

Table 3 Ratios of amplitudes of shallow water to main constituents for TPK and QUB

Ratios	TPK				QUB	
	1973 measured tides	Measured levels for August 1988	TT predicted levels for August 1988	Model simulated levels for August 1988	19-year measured levels	Measured levels for August 1988
M4/M2	0.22	0.36	0.05	0.12	0.09	0.14
M6/M4	0.46	0.28	0.08	0.36	0.1	0.1

Table 4 Comparisons of tide levels generated by different methods for TPK for August 1988

Comparisons between		Errors (m)		
Data Set	Data Set	Maximum	Mean	RMS
Model simulated levels	HA predicted levels	-0.15	0.006	0.094
TT predicted levels	HA predicted levels	0.52	0.066	0.197
Model simulated levels	Measured levels	0.64	0.02	0.166
TT predicted levels	Measured levels	0.81	-0.085	0.258
Measured levels	HA predicted levels	0.51	-0.019	0.138
Model simulated levels	TT predicted levels	0.5	0.103	0.174

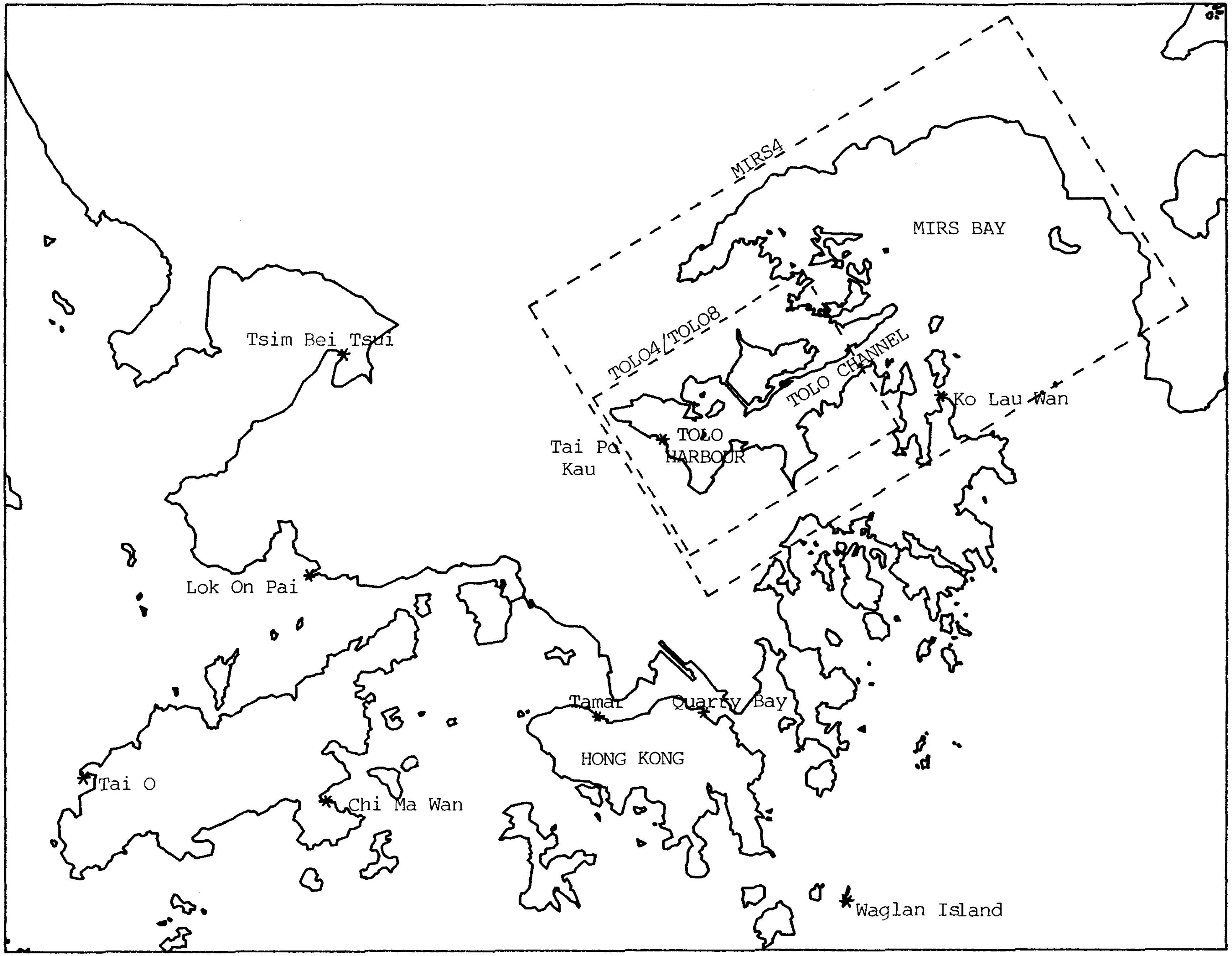

Figure 1 Areas covered by the numerical models (* location of tide gauges)

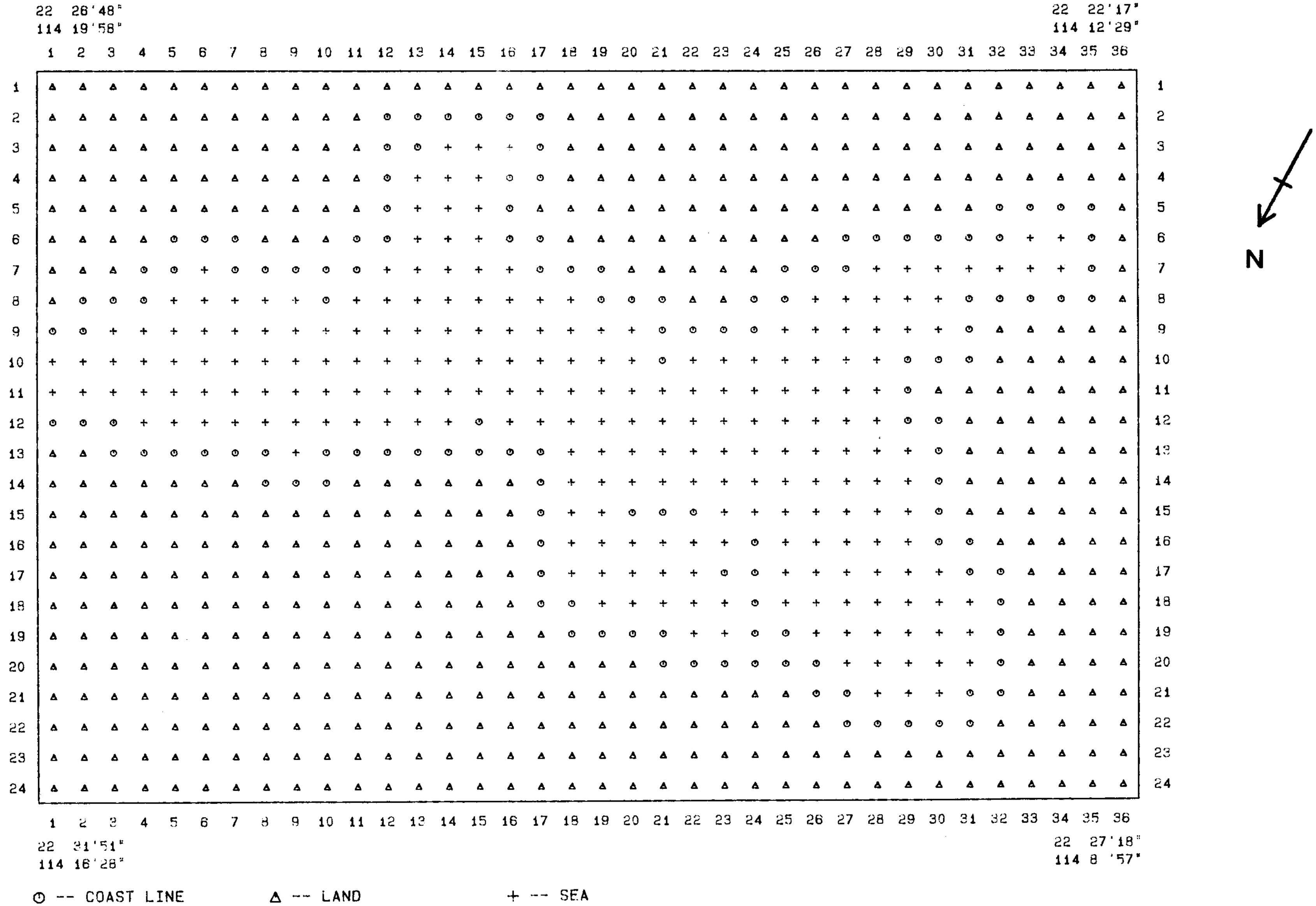

Figure 2 Layout of TOLO4 basin

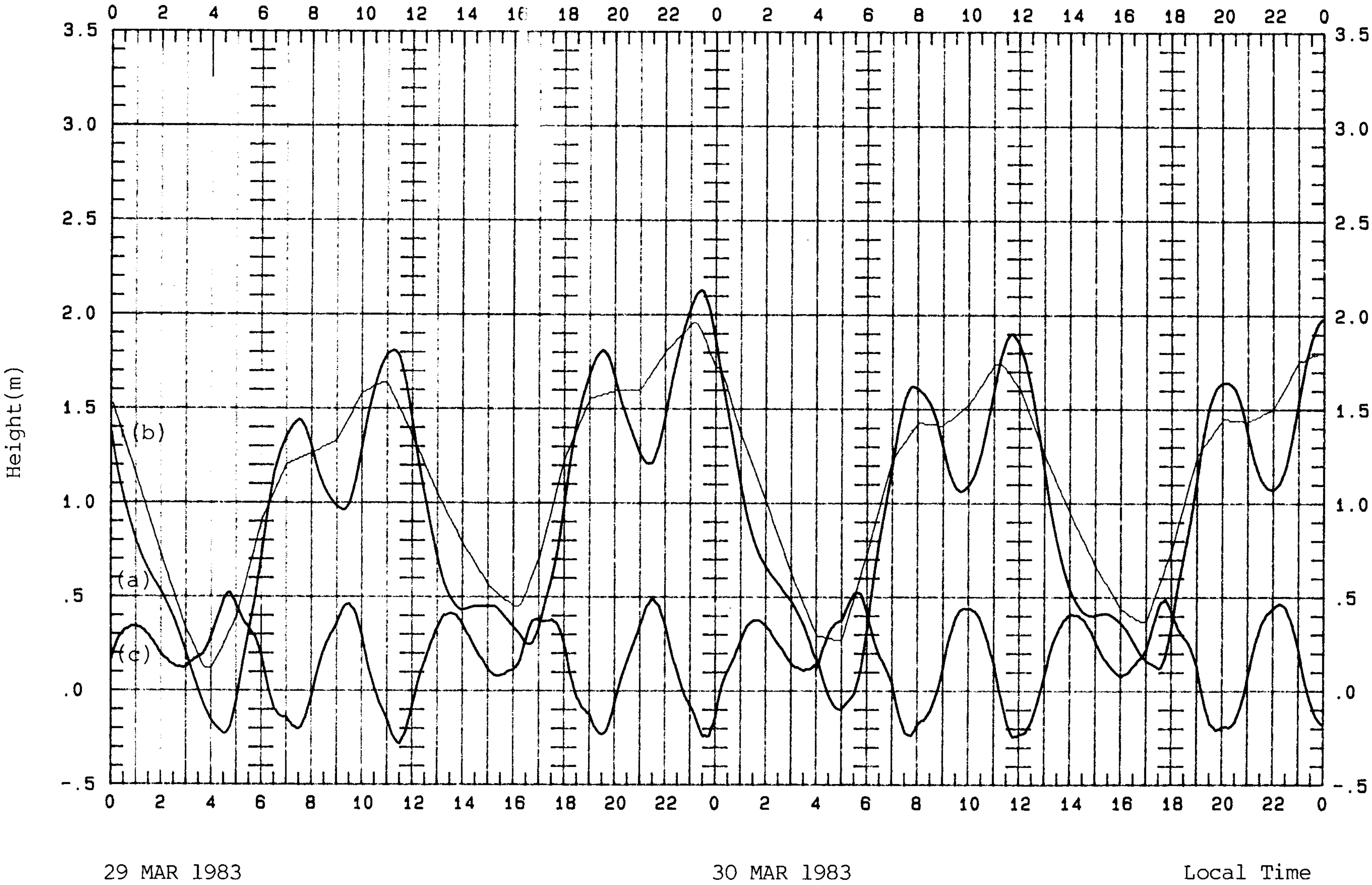

Figure 3 Results of Run Al for TPK (with TOLO4, α=0.98, r=0.003, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

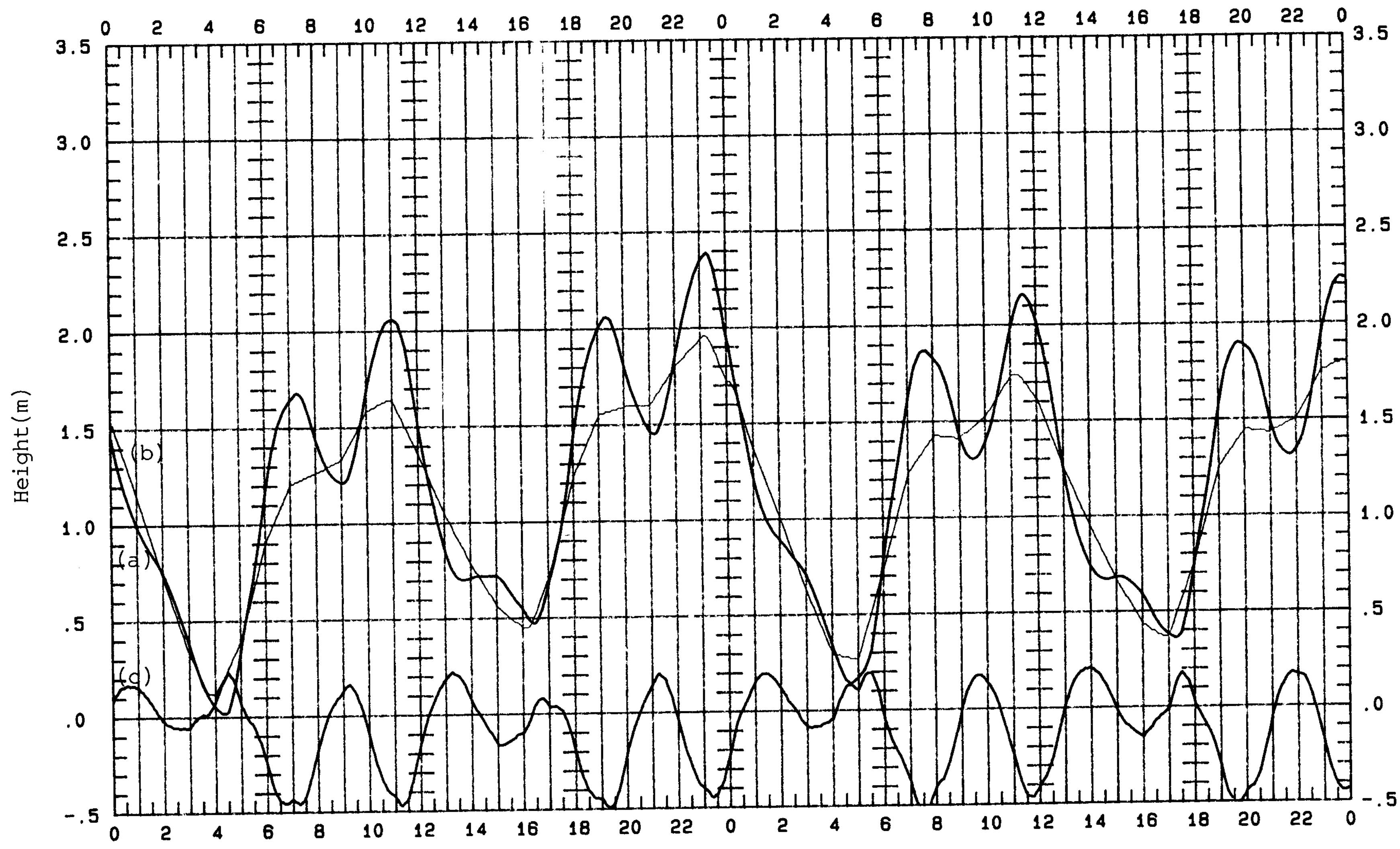

Figure 4 Results of Run A2 for TPK (with TOLO4, α=0.98, r=0.003, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

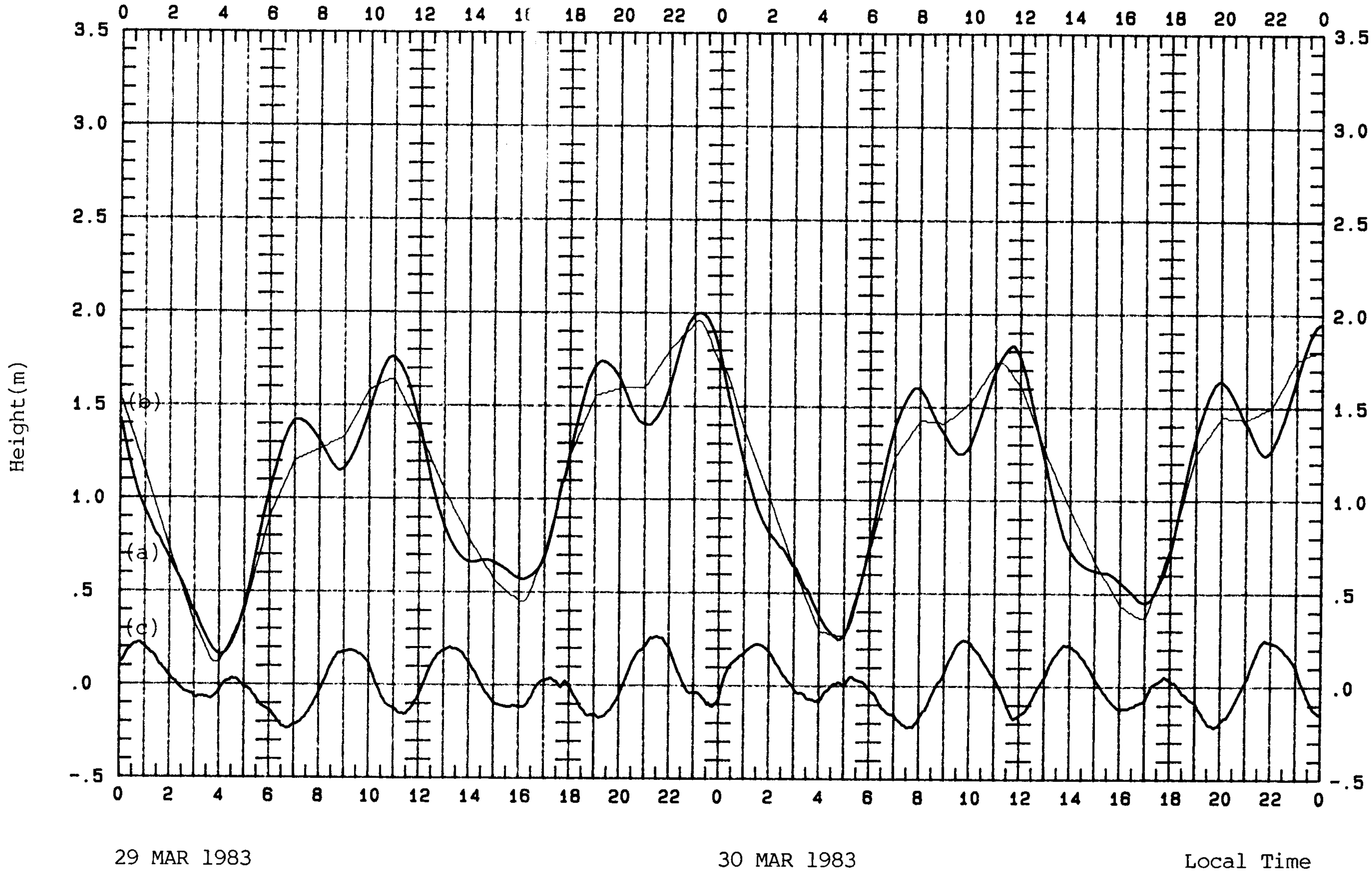

Figure 5 Results of Run A3 for TPK (with TOLO4, α=0.98, r=0.003, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

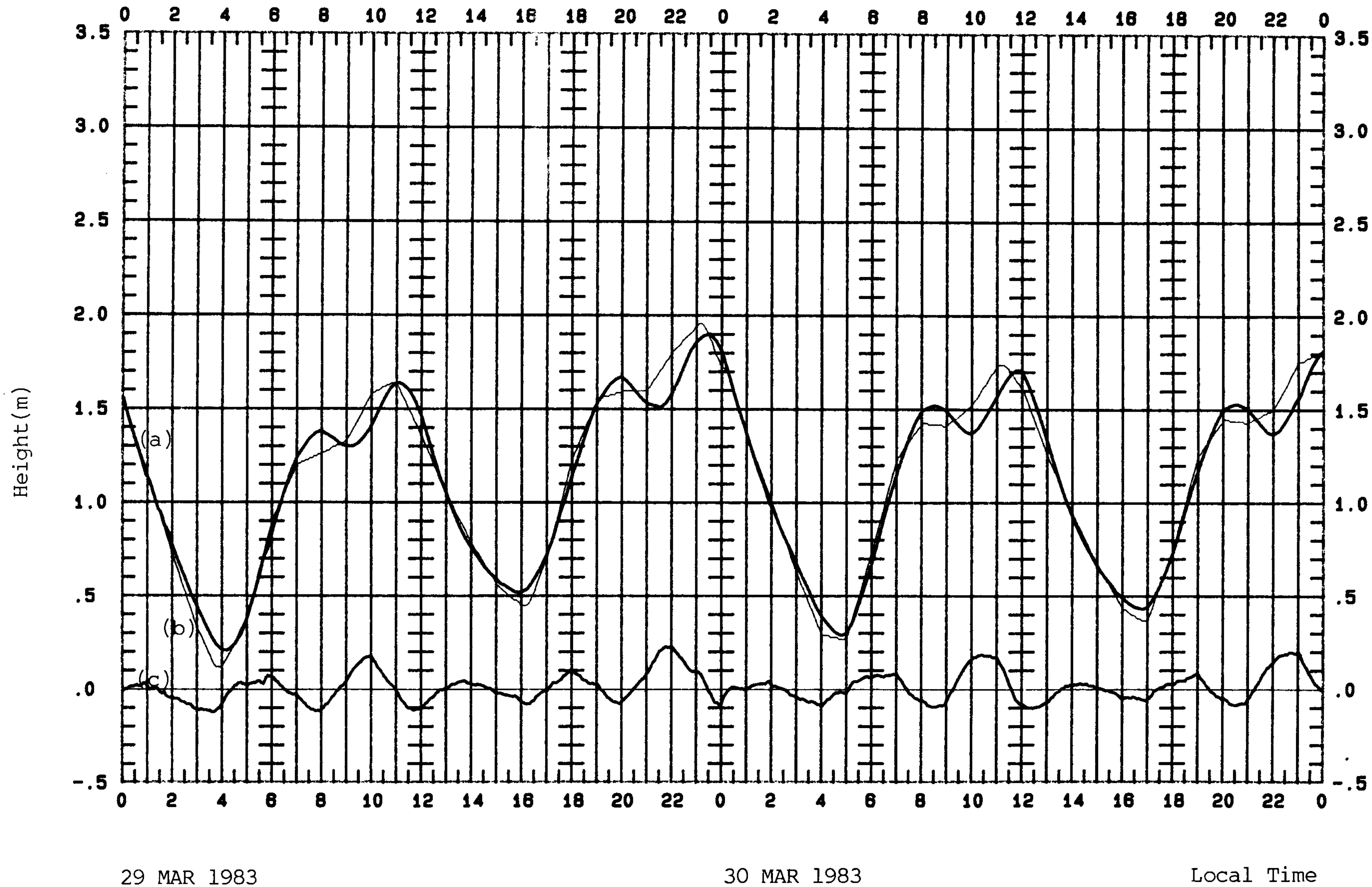

Figure 6 Results of Run A4 for TPK (with TOLO4, α=0.98, r=0.01, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

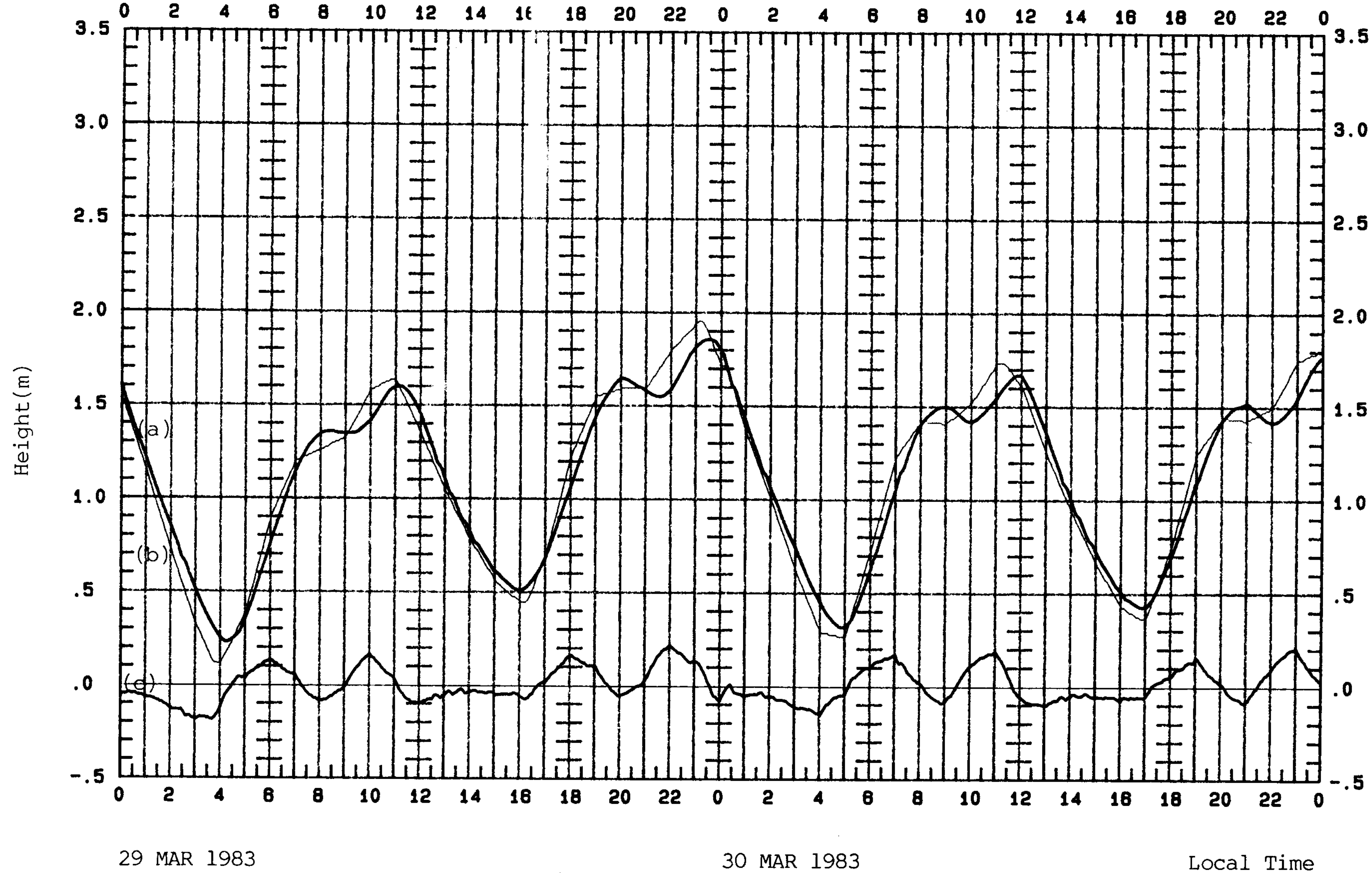

Figure 7 Results of Run A5 for TPK (with TOLO4, α=0.98, r=0.015, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

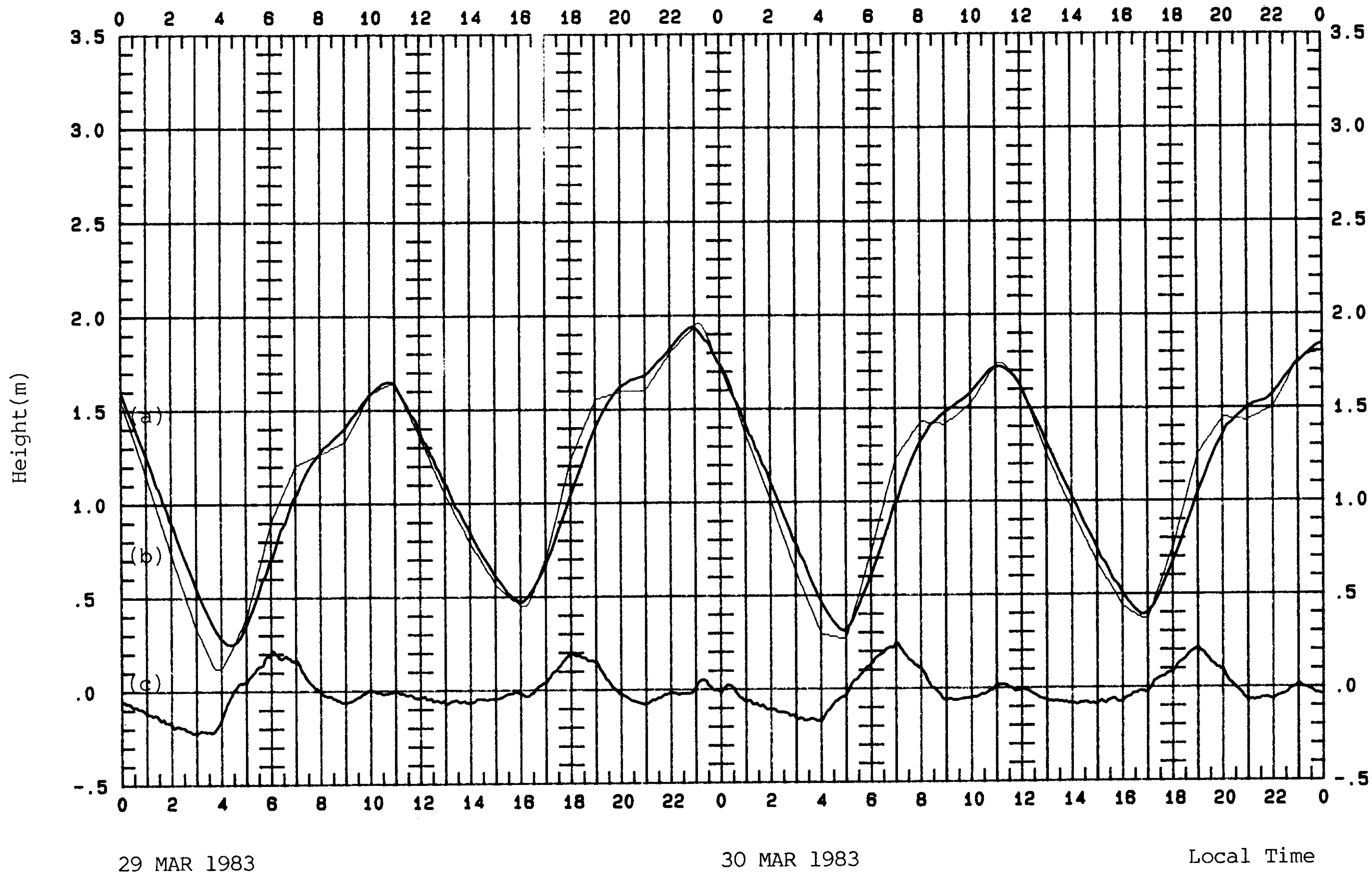

Figure 8 Results of Run A6 for TPK (with TOLO4, α=0.9, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

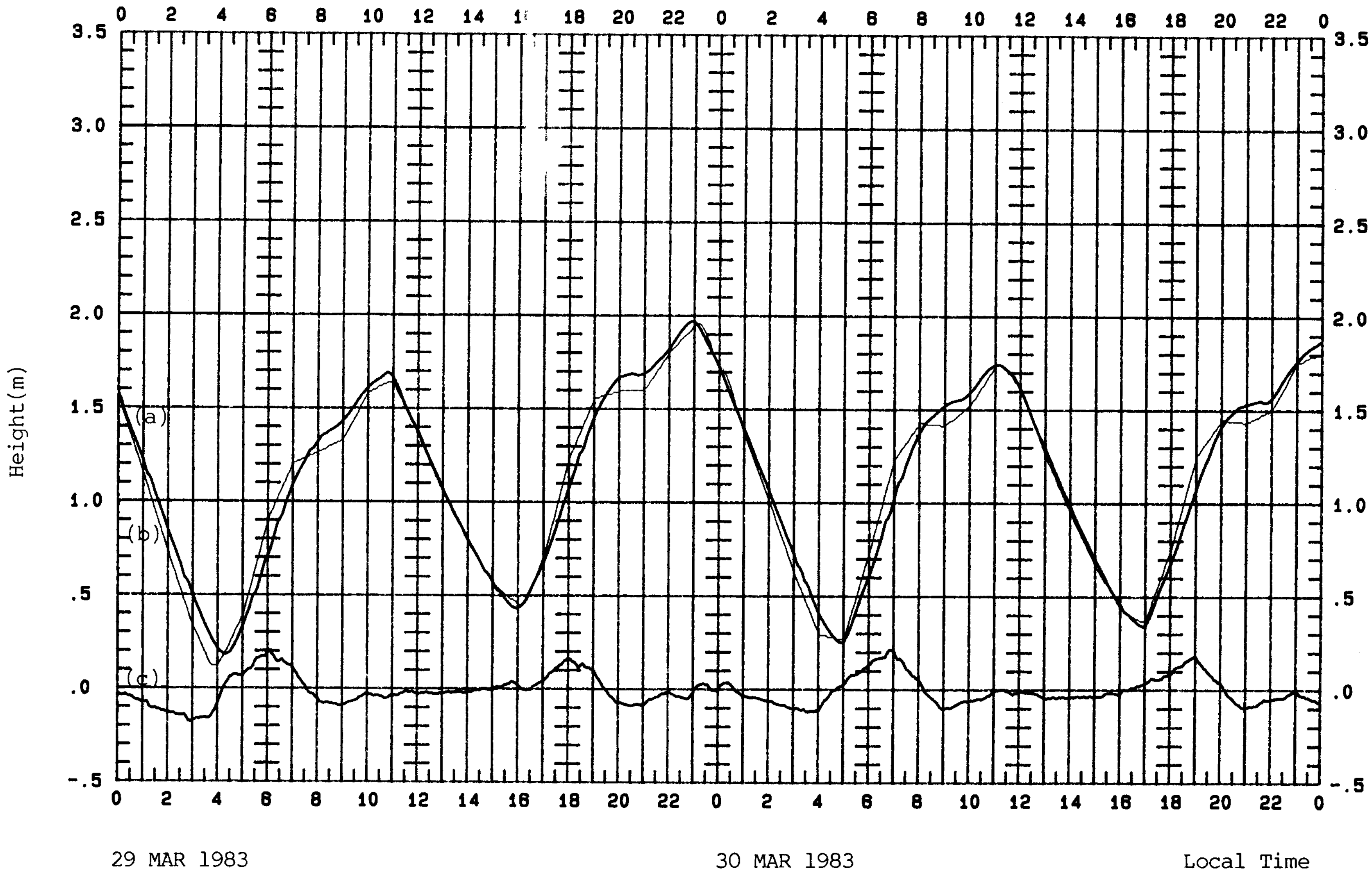

Figure 9 Results of Run A7 for TPK (with TOLO4, α=0.98, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

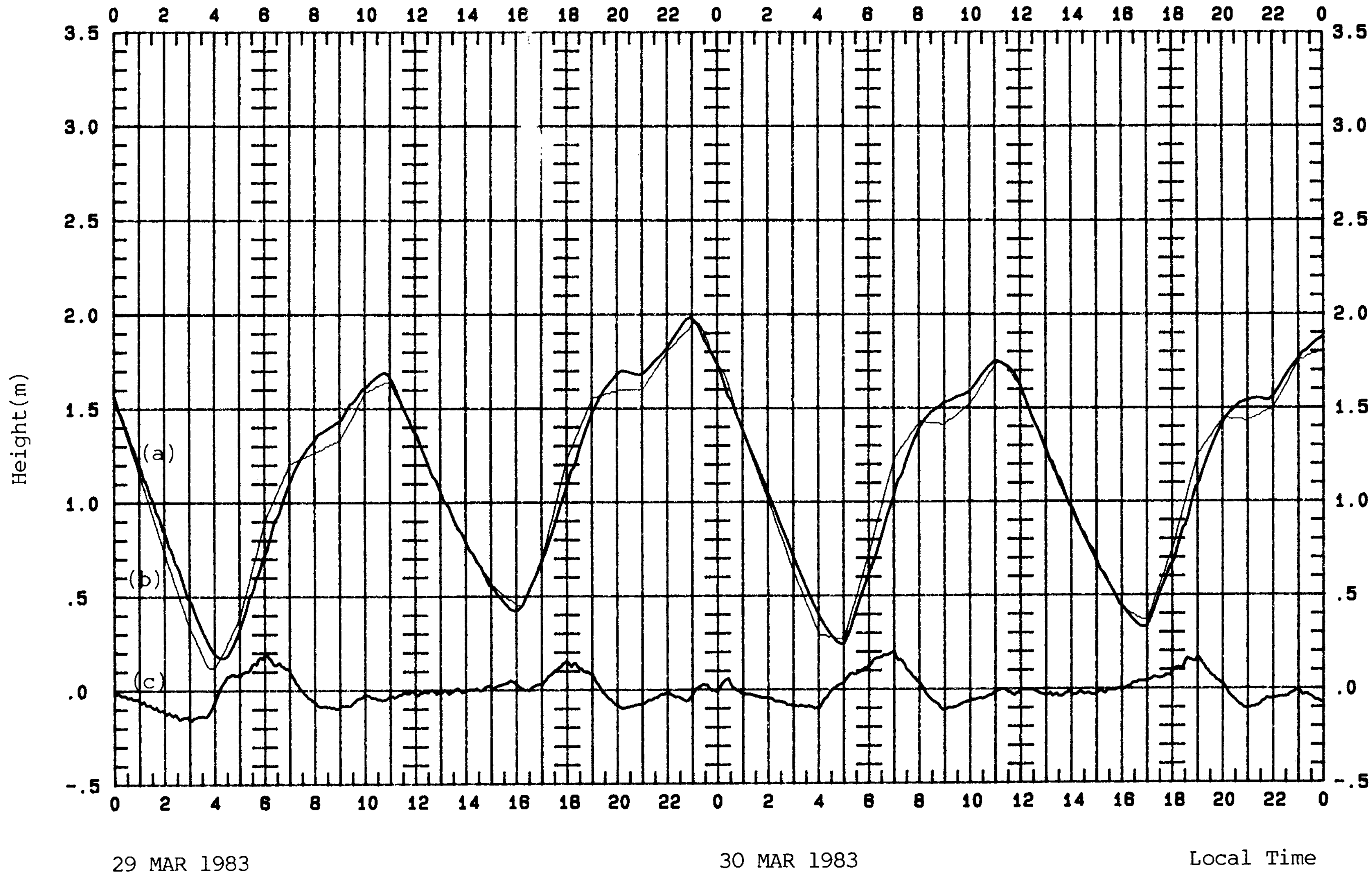

Figure 10 Results of Run A8 for TPK (with TOLO4, α=0.998, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

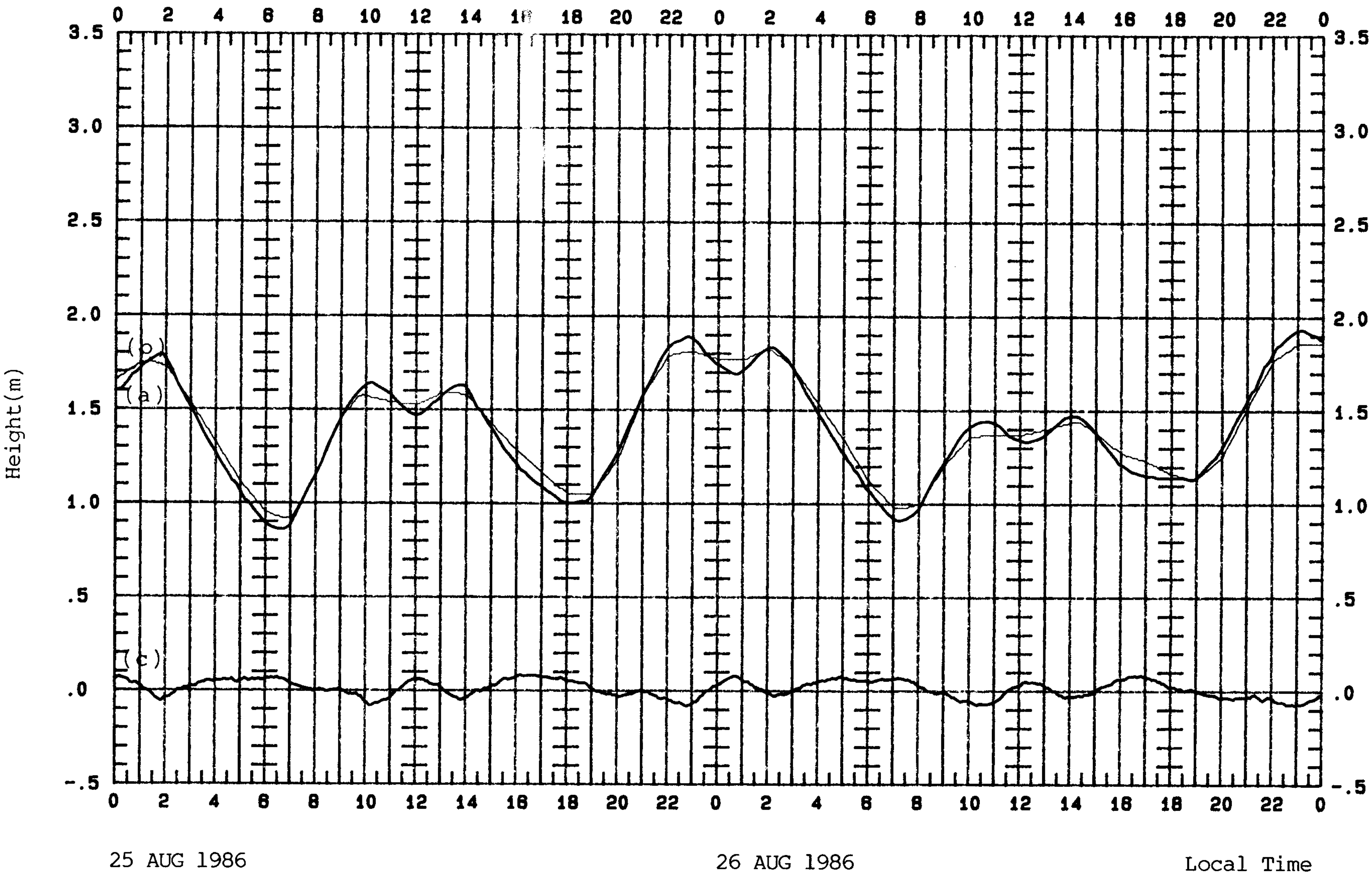

Figure 11 Results of Run A9 for TPK (with TOLO4, α=0.998, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

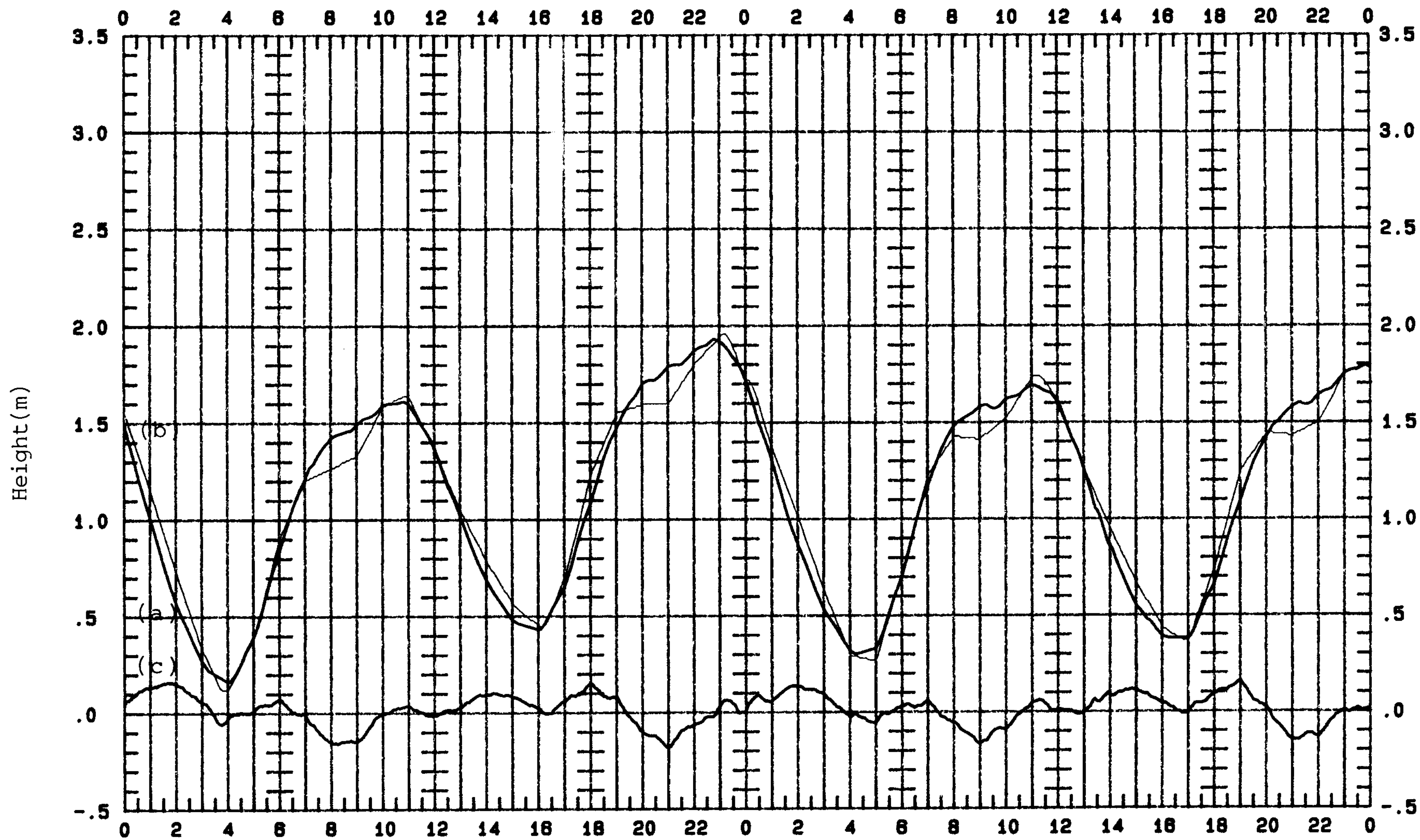

Figure 12 Results of Run A10 for TPK (with TOLO4, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

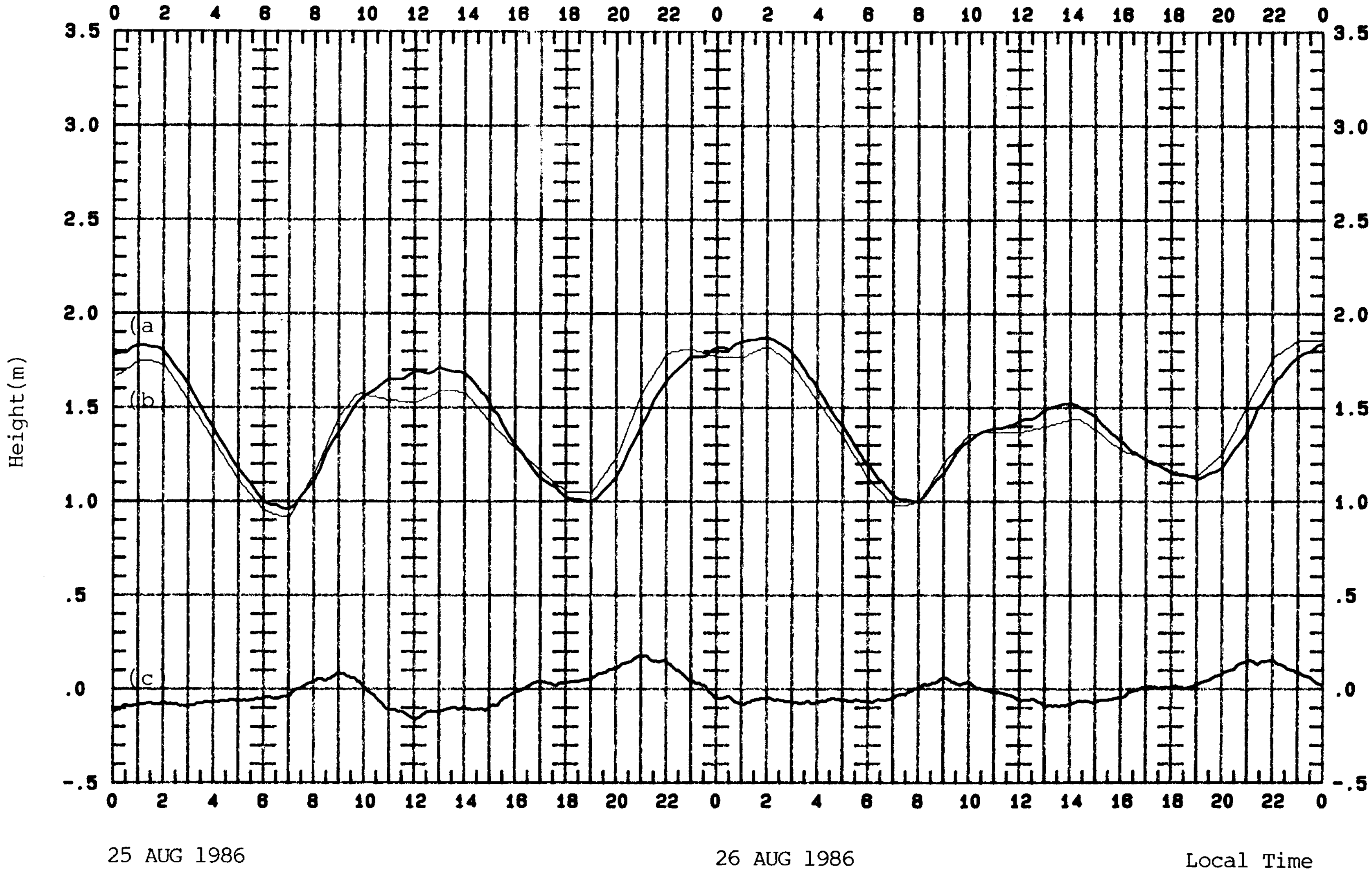

Figure 13 Results of Run All for TPK (with TOLO4, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

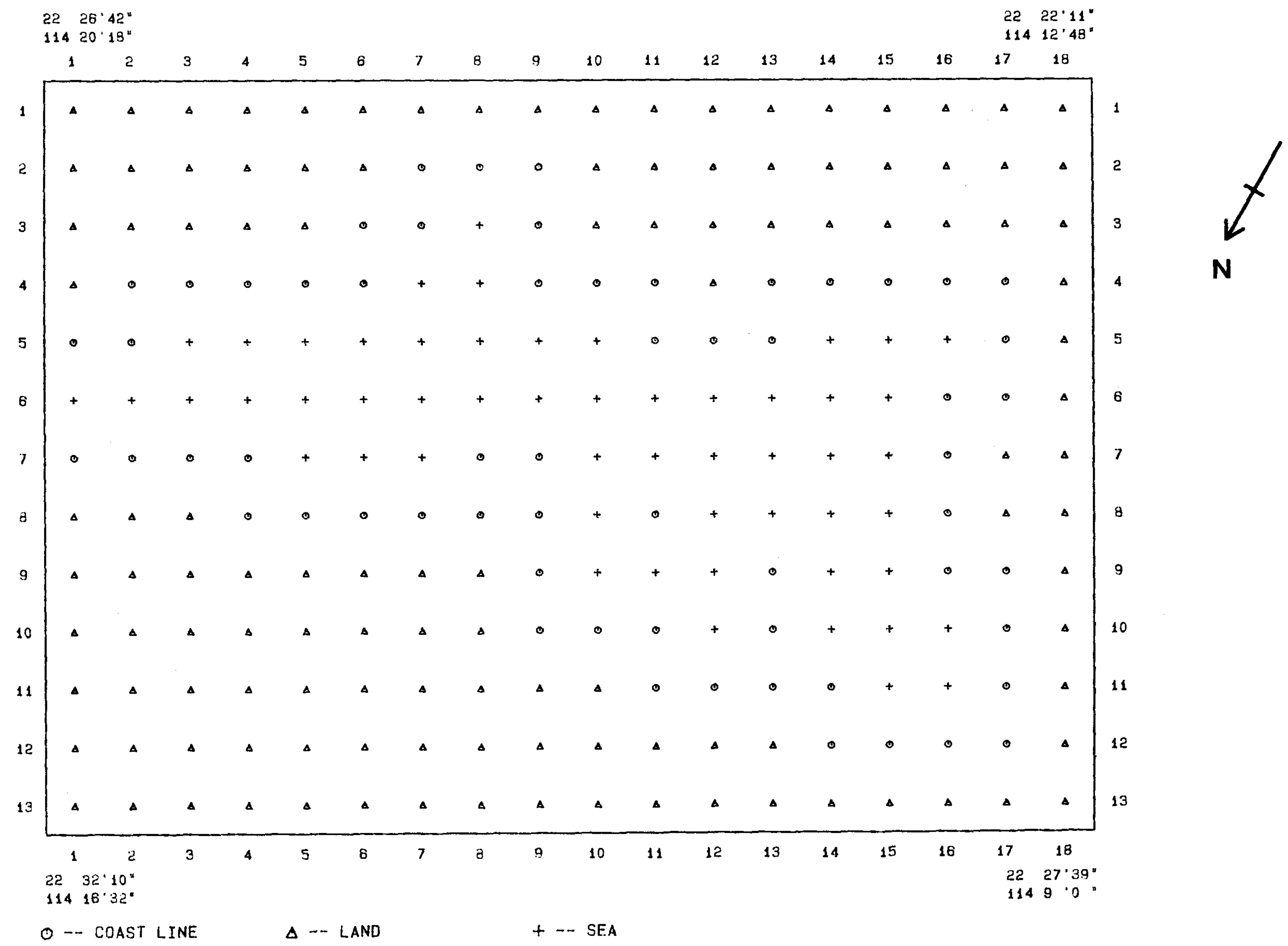

Figure 14 Layout of TOLO8 basin

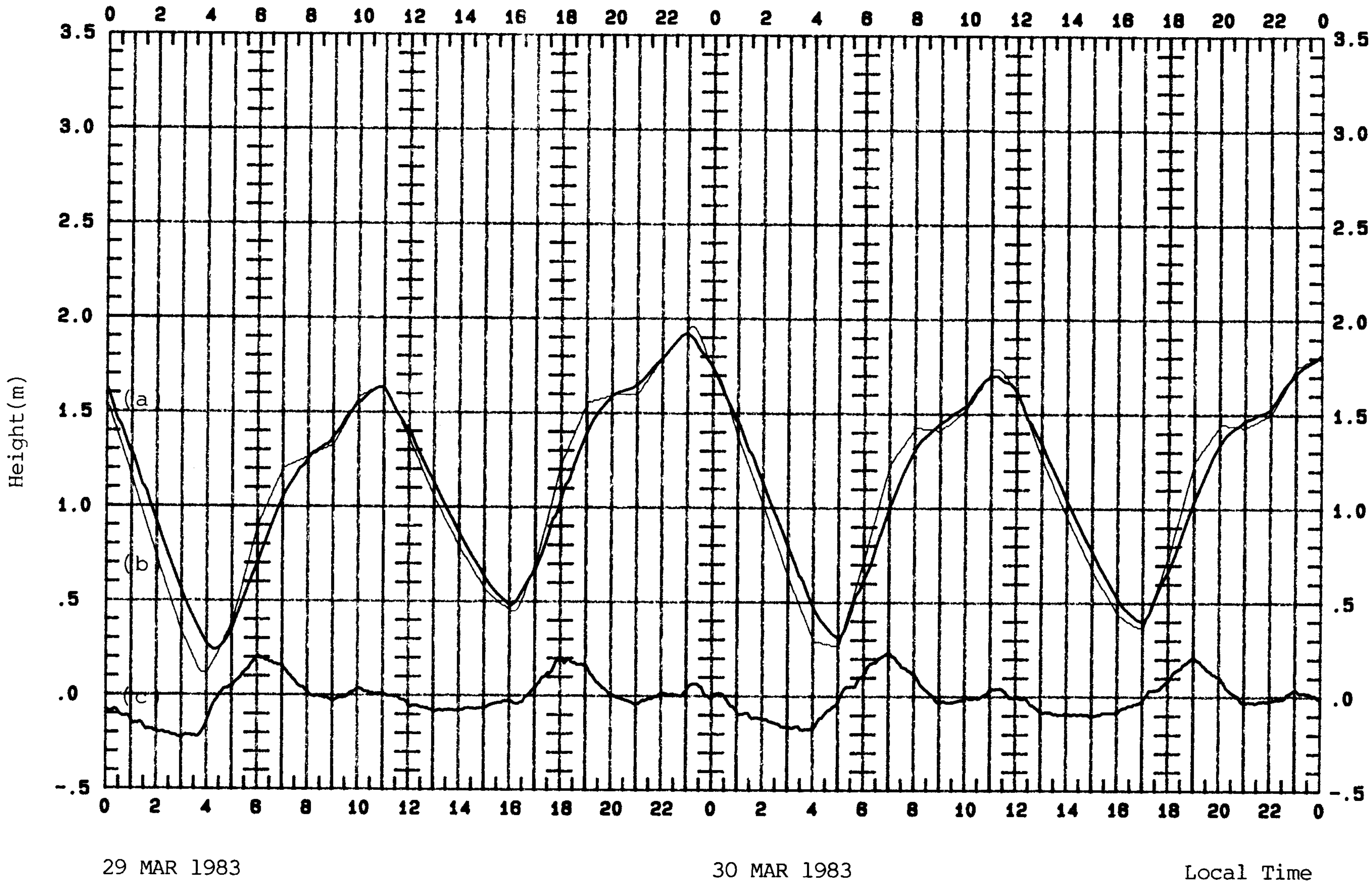

Figure 15 Results of Run B1 for TPK (with TOLO8, α=0.998, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

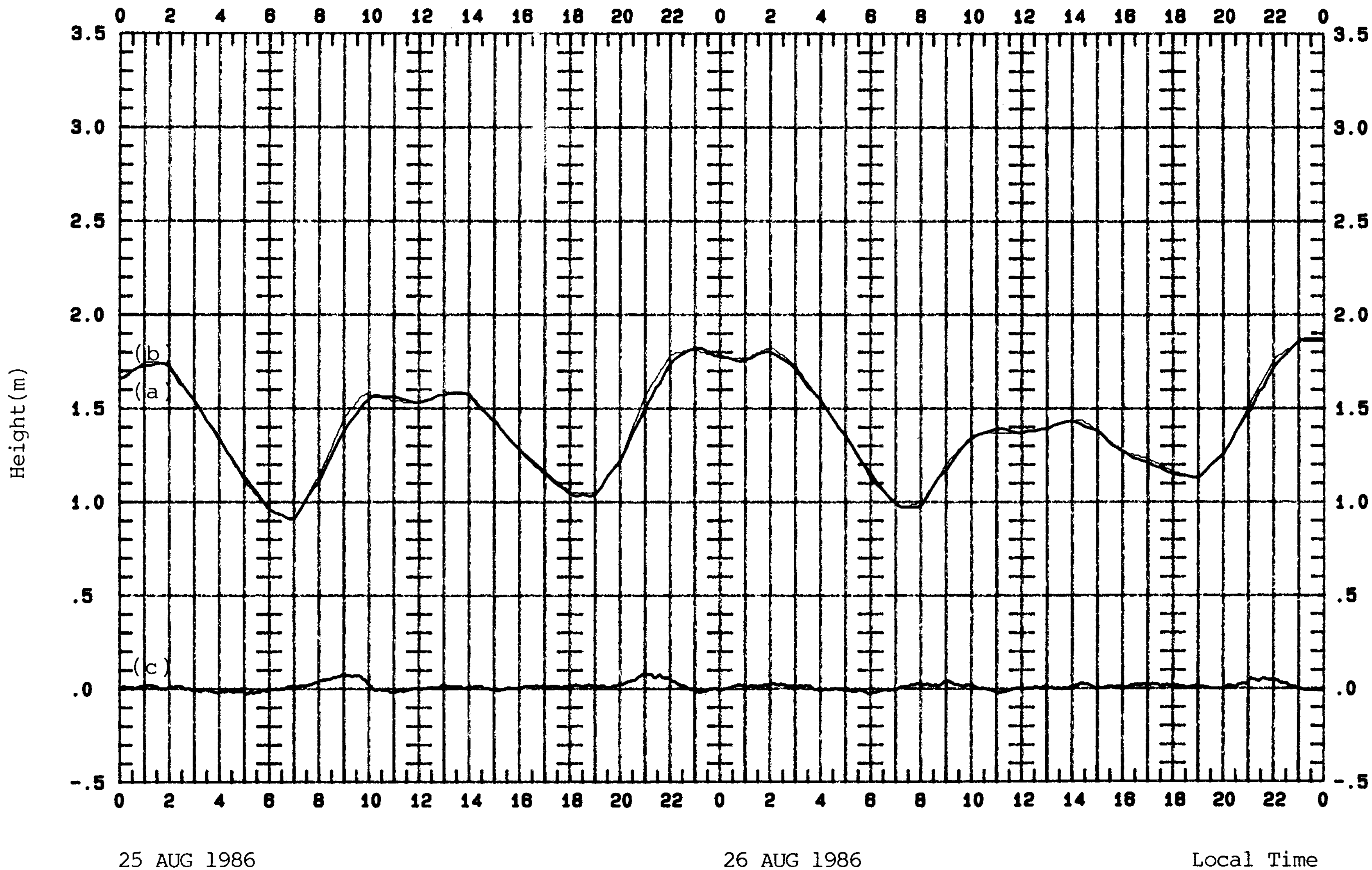

Figure 16 Results of Run B2 for TPK (with TOLO8, α=0.998, r=0.027, TPK as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

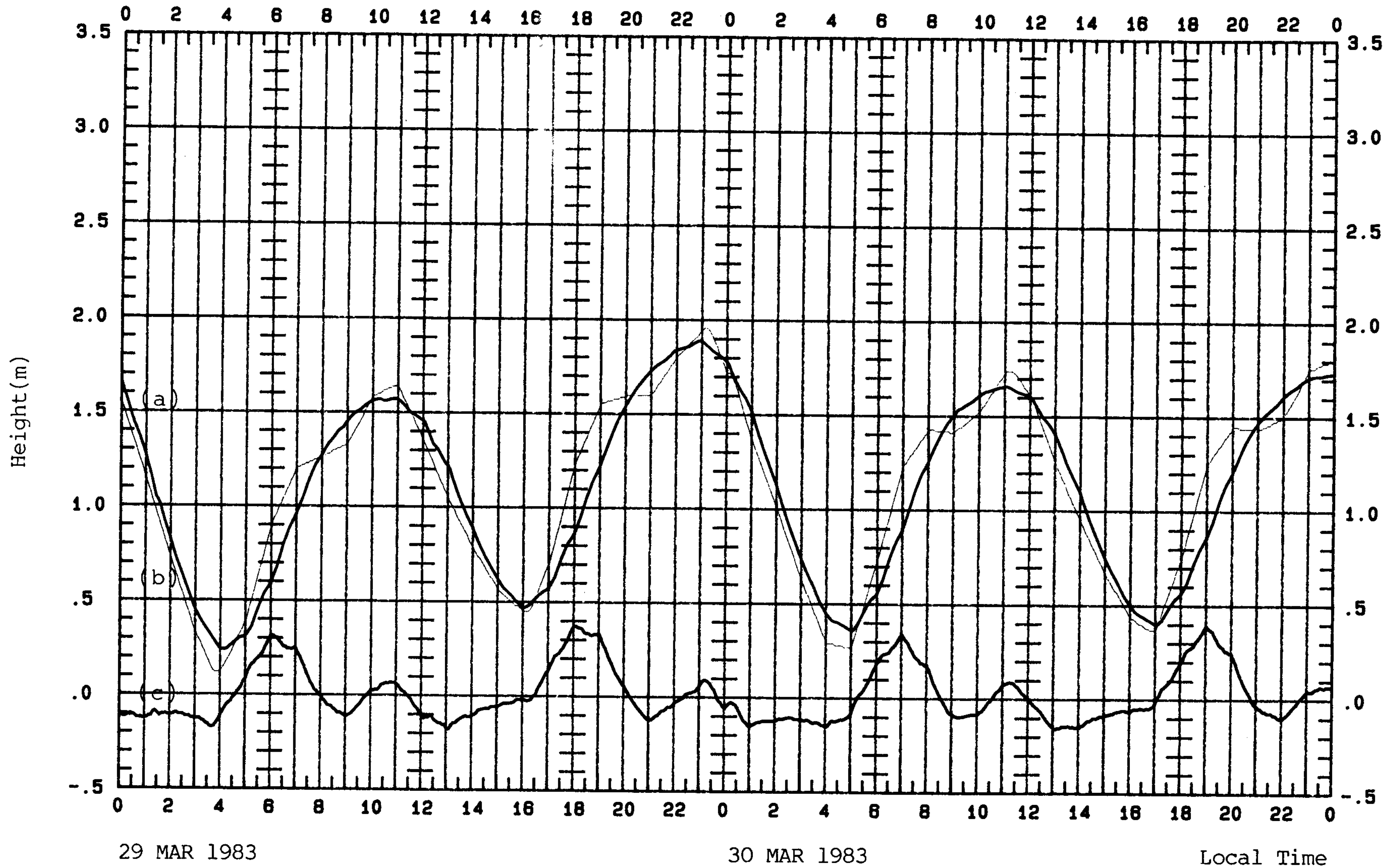

Figure 17 Results of Run B3 for TPK (with TOLO8, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

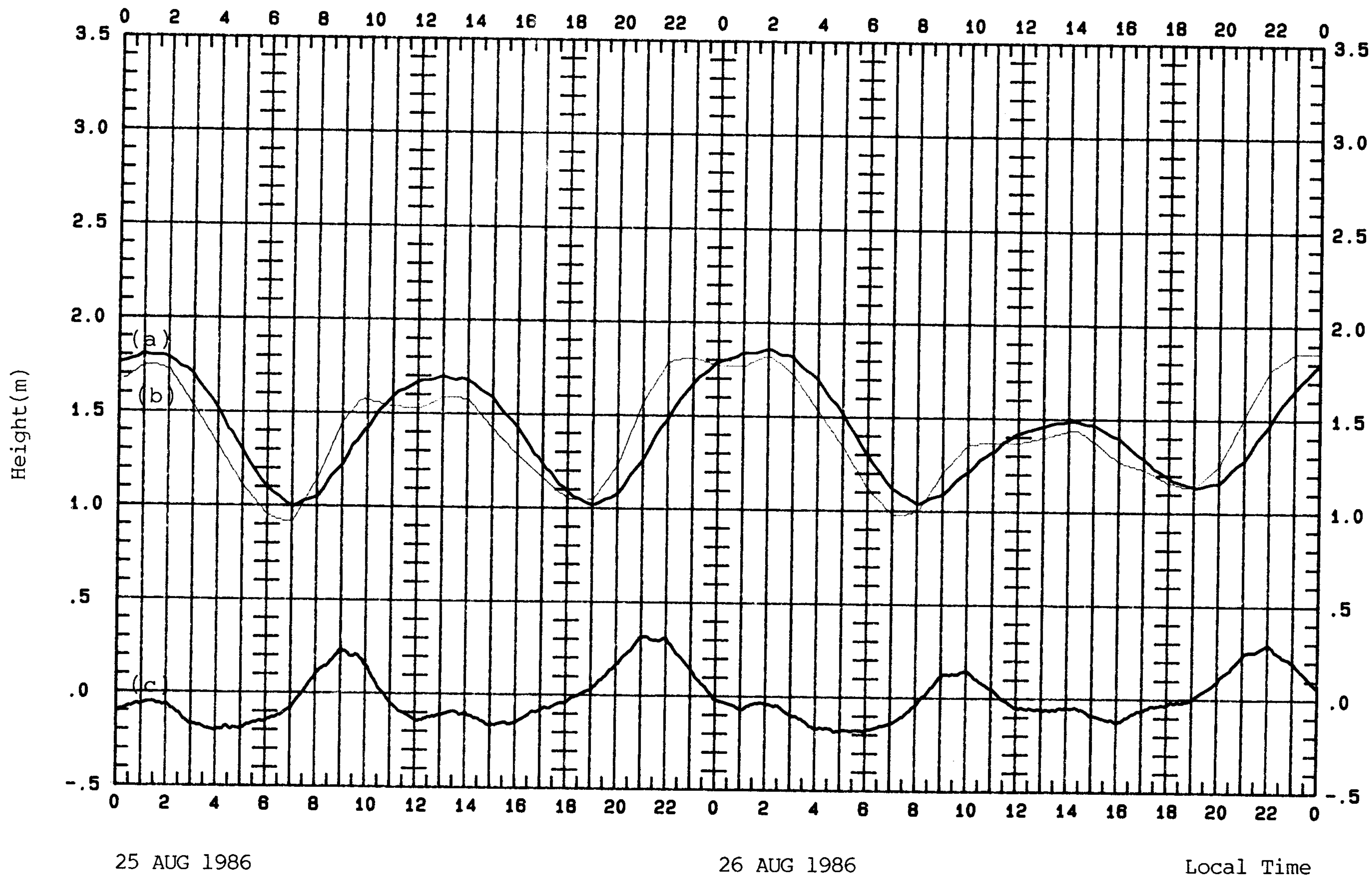

Figure 18 Results of Run B4 for TPK (with TOLO8, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

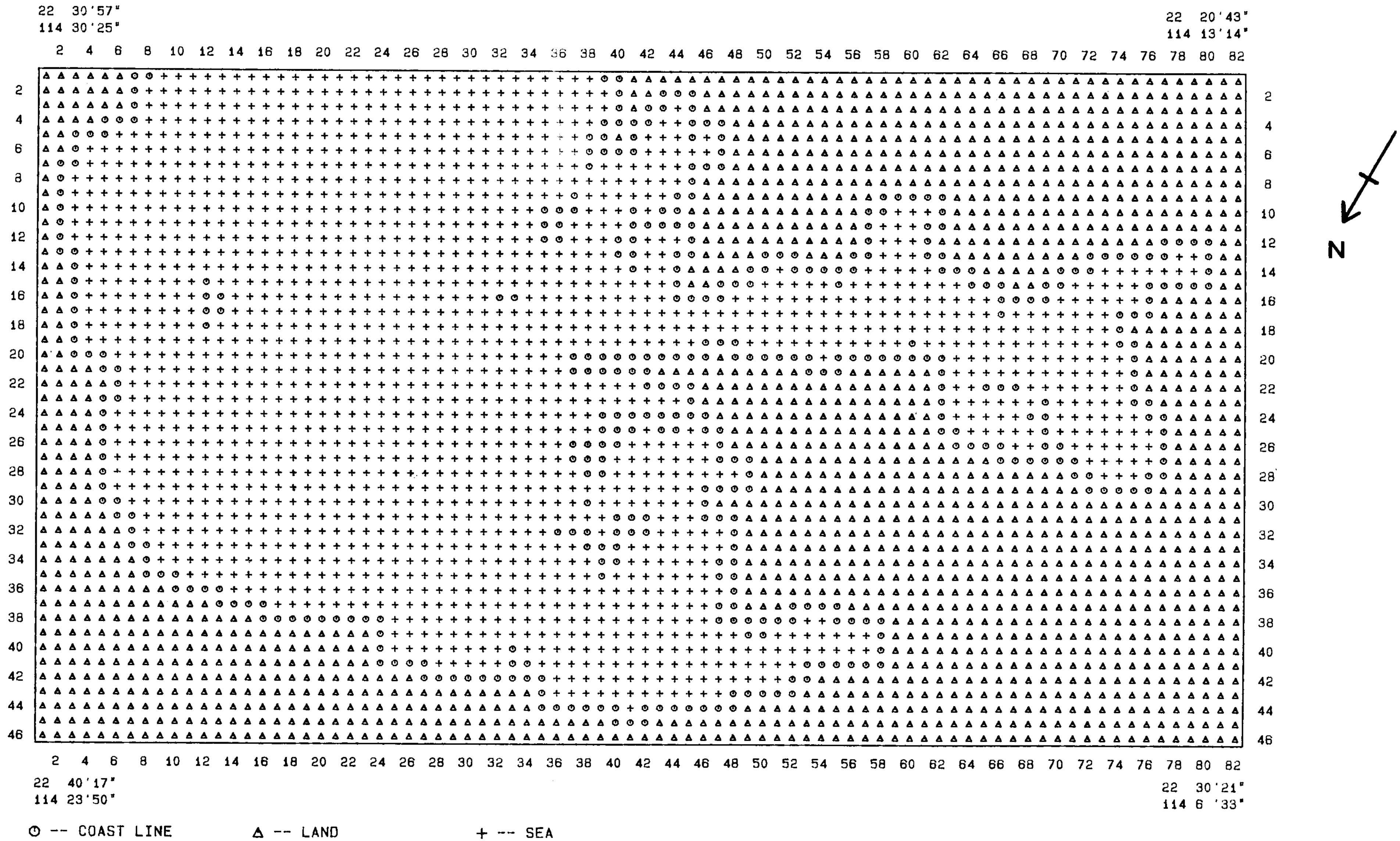

Figure 19 Layout of MIRS4 basin

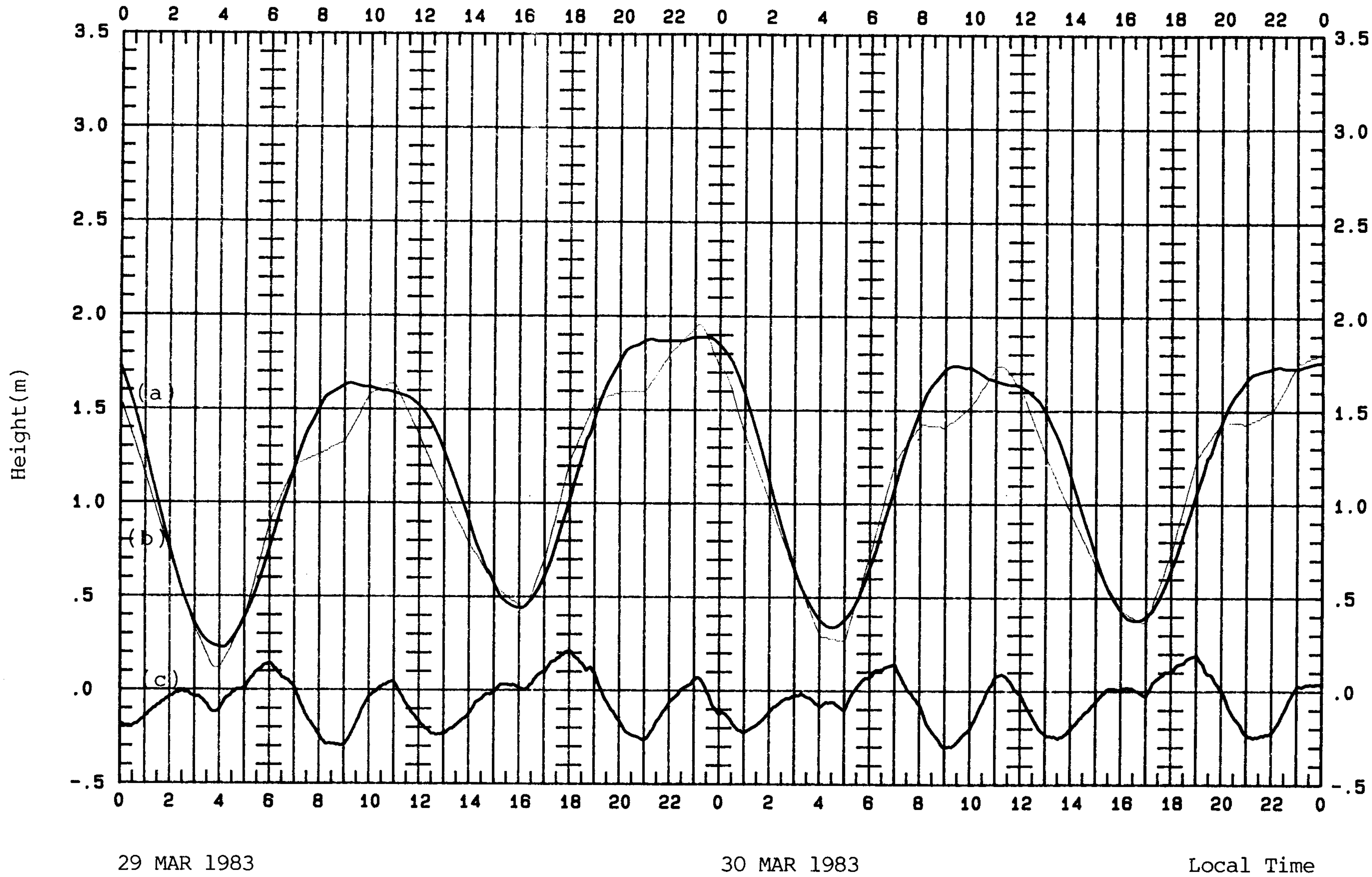

Figure 20 Results of Run Cl for TPK (with MIRS4, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals

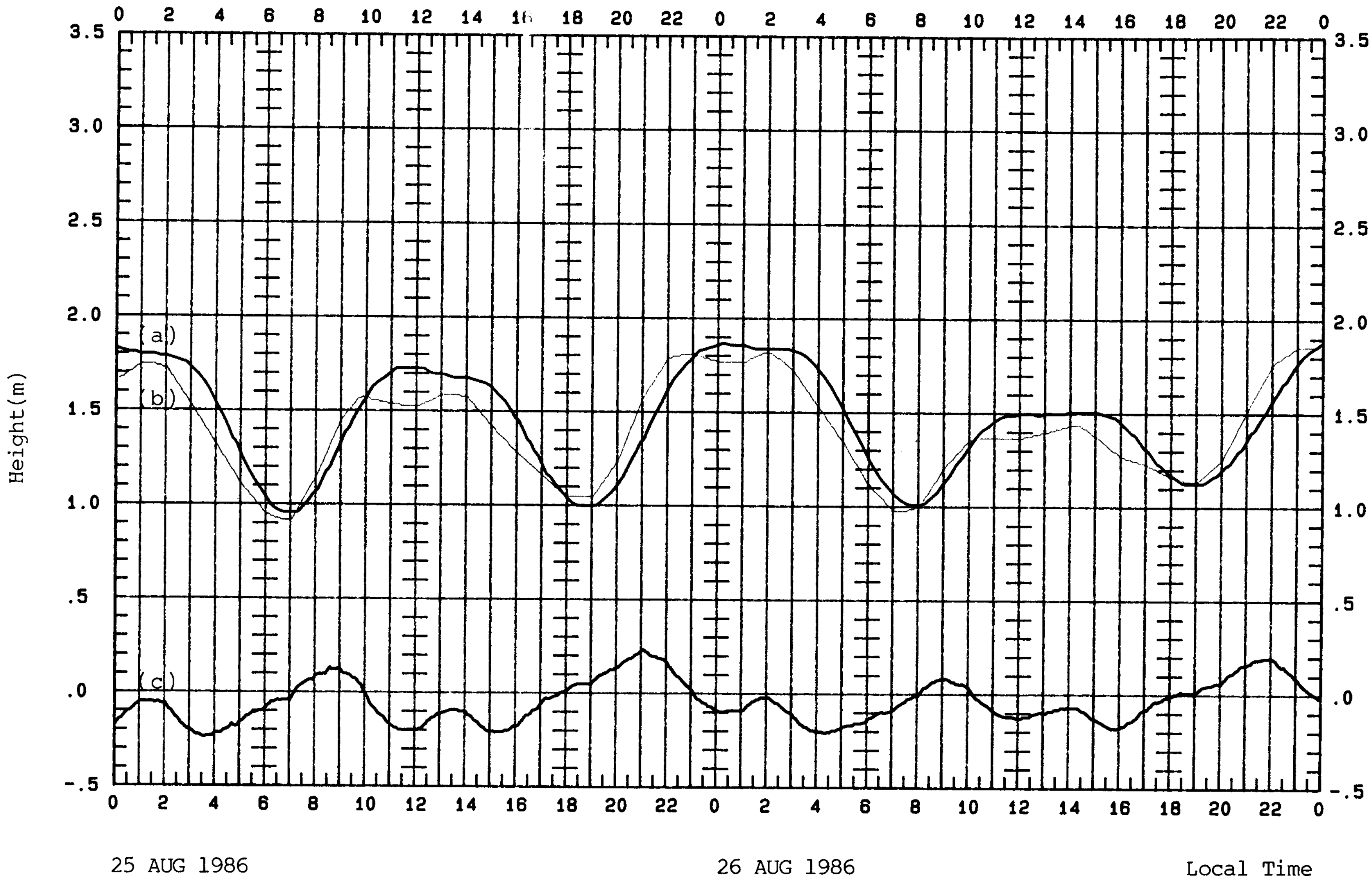

Figure 21 Results of Run C2 for TPK (with MIRS4, α=0.98, r=0.001, QUB as reference for boundary)

(a) model simulated levels (b) expected levels (c) residuals