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COMPARISON OF ALGORITHMS FOR THE COMPUTATION OF THE THERMODYNAMIC PROPERTIES OF MOIST AIR

by

W.T. WONG

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Prepared by

Royal Observatory
134A Nathan Road
Kowloon
Hong Kong

551.501.42

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1. INTRODUCTION

One of the routines in meteorology and weather forecasting is the determination of the moisture content in the atmosphere. There are a number of parameters which measure the amount of water vapour in the atmosphere. Those parameters most frequently encountered in meteorology are listed below:

(a) saturation vapour pressure
(b) mixing ratio
(c) relative humidity
(d) specific humidity
(e) dew-point temperature
(f) wet-bulb temperature
(g) potential temperature of unsaturated moist air
(h) saturation adiabat, equivalent potential temperature, and wet-bulb potential temperature
(i) lifting condensation level

The values of these moisture parameters give a quantitative basis to a forecaster's perception of the wetness of the atmosphere. They are also computed in numerical modelling as well as in the determination of stability indices.

Conventionally, these parameters are obtained either by checking meteorological tables, or using thermodynamic diagrams such as tephigrams. For many applications, it is more convenient to compute these parameters using selected algorithms or formulae. The purpose of this technical note is to summarize different algorithms and to evaluate their accuracy and relative speeds of computation.

In section 2, the basic concepts, methods or algorithms of computation and reference data for the above moisture related parameters are described briefly. Section 3 presents the results of a series of comparisons and makes recommendation on the choice of algorithms for operational applications.

2. MOISTURE PARAMETERS

(a) Saturation vapour pressure

Concept:

A parcel of air becomes saturated with water vapour when the water vapour is in equilibrium with the liquid phase or solid phase. The saturation vapour pressure over a plane surface of pure water is a function of the temperature only and can be calculated by integrating the Clausius-Clapeyron equation.

Due to the effect of surface tension, impurity of the liquid or solid phase, and deviation from the ideal gas law, the saturation vapour pressure of moist air is slightly different from the saturation vapour pressure over a plane surface of pure water or pure ice. The deviation is however, quite small, and is less than 0.05 % in the range of temperature and pressure encountered in meteorology (WMO 1966).

Method of Computation:

The saturation vapour pressure over a plane surface of pure water or pure ice is obtained by integrating the Clausius-Clapeyron equation

$$\frac{de}{dT} = \frac{\gamma}{T\nu} \qquad (1)$$

for which γ is the latent heat of vaporization and ν the specific volume at temperature T. The relationship of γ and ν with temperature T is measured in calorimetric experiments and use is made of the virial equation

$$e\nu = R_w T - eA_1(T) - e^2 A_2(T) - \ldots \qquad (2)$$

Three formulae for the saturation vapour pressure over water have been obtained from the procedures described above. The first two were due to Goff and Gratch (1946 & 1949). The saturation vapour pressure tabulated in the Smithsonian Meteorological Tables (List 1963) and the International Meteorological Tables (WMO 1966) were calculated respectively from these two formulae. The third one is due to Wexler (1976) and takes into account the data measured in the early 70's.

Two corresponding formulae for the saturation vapour pressure over ice due to Goff and Gratch were used for the generation of the values tabulated in the Smithsonian Meteorological Tables and the International Meteorological Tables.

Other formulae for the calculation of saturated vapour pressure are obtained either by assuming the latent heat of vaporization γ to be a constant and using the ideal gas law for the specific volume in integrating the Clausius-Clapeyron equation, or by numerically fitting the values tabulated in the Smithsonian Meteorological Tables or International Meteorological Tables.

(b) Mixing ratio

Concept:

The mixing ratio r of moist air, defined as the ratio of the mass of water vapour to that of dry air with which the water vapour is associated, is a measure of the relative number of the vapour mole fraction to the dry air mole fraction.

Method of Computation:

The mixing ratio is calculated from the vapour pressure and the atmospheric pressure using the formula

$$r = \varepsilon \frac{e}{p - e} \tag{3}$$

where $\varepsilon = 0.62198$ is the ratio of the molar mass of water vapour to that of dry air.

Due to deviation from the ideal gas law, the vapour pressure used in the equation must be adjusted in order to evaluate the mixing ratio of air. This is accomplished by multiplying the vapour pressure in equation (3) an adjustment coefficient f and we have

$$r = \varepsilon \frac{fe}{p - fe} \tag{3a}$$

where f depends on the temperature and pressure of the air parcel (table 89, List 1963).

(c) Relative humidity

Concept:

The relative humidity R.H. with respect to water of moist air at pressure p and temperature T, is the ratio in per cent of the vapour mole fraction to the vapour mole fraction which the air would have if it were saturated with respect to water at the same pressure and temperature. In other words, the relative humidity measures the amount of water vapour present relative to that allowed at the same temperature and pressure.

Method of Computation:

The relative humidity can be calculated from the vapour pressure or the dew-point of the air parcel, and the temperature of the parcel using the formula

$$R.H. = \frac{e}{e_s(T)} \times 100\ \% = \frac{e_s(T_d)}{e_s(T)} \times 100\ \% \qquad (4)$$

or directly measured by a humidity meter which is used in upper-air soundings.

(d) Specific humidity

Concept:

The specific humidity S.H. of moist air is the ratio of the mass of water vapour to the mass of the moist air (including dry air and water vapour) in which the mass of water vapour is contained. It is related to the mixing ratio by

$$S.H. = \frac{r}{1 + r} \qquad (5)$$

(e) Dew-point temperature

Concept:

Dew point T_d is the temperature at which the moist air will become saturated with the same mixing ratio and at the same pressure. Alternatively, the dew-point may be considered as the temperature at which the vapour pressure of the air will become saturated i.e. $e = e_s(T_d)$.

Method of Deduction:

The dew-point T_d is calculated from the amount of water vapour in the air by considering the vapour pressure as the saturation vapour pressure at T_d. The vapour pressure of the parcel of air in question is, in turn, computed from the observed temperature, wet-bulb temperature and pressure, or from relative humidity and temperature which are measured in upper air soundings.

(f) Wet-bulb temperature

Concept:

The thermodynamic wet-bulb temperature T_w of moist air at temperature T, pressure p and mixing ratio r is defined as the temperature T_w attained by the moist air when brought adiabatically to saturation at the same pressure p by evaporation into the moist air, of liquid water at pressure p and temperature T_w, and containing the amount of dissolved air corresponding to equilibrium with saturated air of the same pressure and temperature (WMO 1979).

The thermodynamic wet-bulb temperature defined above is obtained in an adiabatic isobaric process, while the pseudo wet-bulb temperature, which as defined by Normand's rule, is the temperature where the saturation adiabat through the lifting condensation level crosses the pressure p of the moist air, is obtained through an adiabatic non-isobaric process. Due to the difference in the work done against the external pressure in the two processes, the thermodynamic wet-bulb temperature is slightly higher than the pseudo wet-bulb temperature. This difference is, however, very small and can be neglected in all meteorological applications (Shen 1983).

Method of Measurement:

The wet-bulb temperature is measured by embracing the bulb of a thermometer by a piece of wet cloth part of which is immersed in a small pool of water.

Relation with Vapour Pressure:

Considering the enthalpy changes in the adiabatic processes and regarding air and vapour as ideal gases with constant specific heat capacities, the thermodynamic wet-bulb temperature can be shown to obey

$$T - T_w = \frac{[r(T_w) - r(T)]\ L}{C_p - r\,C_{pv}}. \qquad (6)$$

If we express the mixing ratio in terms of the ratio of partial pressures and noting that the vapour pressure is small compared with the total pressure, we have

$$e \cong e_s(T_w) - \frac{1}{\varepsilon}\,p\,\frac{C_p - rC_{pv}}{L}\,(T - T_w) \qquad (7)$$

or

$$e \cong e_s(T_w) - A\,p\,(T - T_w) \qquad (7a)$$

In practical situations, due to the effects of convection and diffusion, the measured wet-bulb temperature may deviate from the value defined above. To compute the vapour pressure from equation (7a) and the measured values of the wet-bulb temperature, the coefficient A must be adjusted to take into account these effects. The Smithsonian Meteorological Tables suggests a value of 0.000660 * (1 + 0.00115 $\mathbf{T_w}$) for the coefficient A where here $\mathbf{T_w}$ is measured in Kelvin. The Hygrometric Tables (UK Met. Office 1964) recommends a value of 0.799 x 10^{-3} for a ventilation of 1 to 1.5 ms^{-1} and 濕度查算表 (中央氣象局 1980) a value of 0.667 x 10^{-3} for a ventilation of 3.5 ms^{-1}. The Royal Observatory uses the value 0.799 x 10^{-3} for the computation of vapour pressure from the wet-bulb temperature.

(g) Potential temperature of unsaturated moist air

Concept:

A parcel of unsaturated moist air, when lifted adiabatically will behave just like a parcel of dry air. But due to the presence of water vapour, the gas constant and the specific heat capacity at constant pressure and hence the lapse rate, will be different from those of dry air.

Method of Computation:

The gas constant and the specific heat capacity for moist air are related to those of dry air through

$$R_m = R_d \; (1 + 0.608\text{x}10^{-3} \; r)$$

$$C_{pm} = C_{pd}(p,T) \; (1 + 0.887\text{x}10^{-3} \; r).$$

Therefore the lapse rate is given by

$$\Gamma_m = R_m/C_{pm} \simeq R_d/C_{pd} \; (1 - 0.279\text{x}10^{-3} \; r). \qquad (8)$$

For the range of pressure and temperature encountered in meteorology, the deviation of Γ_m from the dry adiabatic lapse rate Γ is very small. For example, even if the mixing ratio is 30 g/kg, a value which is exceptionally high and occurs only rarely near the surface of the tropical ocean, the deviation is about 0.837 % only. As a result the above adiabatic process can be very well approximated by the dry adiabat. The potential temperature is represented to a high degree of precision by the Poisson's equation

$$\theta = T \left(\frac{1000}{p}\right)^{R_d/C_{pd}} \qquad (9)$$

(h) Saturation adiabat, equivalent potential temperature and wet-bulb potential temperature

Concept:

Saturation adiabat is the line on a thermodynamic diagram, along which a parcel of air saturated with water vapour changes adiabatically. Upon up-lifting of the air parcel, if the water that condensed out remains in the parcel, the process is reversible and the entropy is conserved. Otherwise the process is irreversible and is oftenly referred as pseudoadiabatic.

Both reversible and irreversible processes are present in nature for changes along saturation adiabats. But the irreversible "pseudoadiabat" is used for the saturation adiabat in all calculation due to its relatively simpler formula and the fact that the differences between the two curves are very small (Yang et al 1983, Shen 1983, Wallace and Hobb 1977).

The equivalent potential temperature θ_e is the temperature where the asymptote of the saturation adiabat on the tephigram cuts the 1000 hPa level and the wet-bulb potential temperature θ_w is the temperature on the saturation adiabat at the 1000 hPa level. Any one saturation adiabat is defined by the θ_e or θ_w and there is a one-to-one correspondence between the two **potential** temperatures.

Mathematically the pseudoadiabat is defined by the differential equation

$$(C_p + r_w C_w)\, d(\ln T) - R\, d(\ln(p-e_w)) + d(L_v r_w / T) = 0 \qquad (10)$$

where the first term describes the heat absorbed by the air parcel due to temperature change, the second term is the work done against external pressure in expansion and the last one accounts for the latent heat released during water condensation.

Algorithms: There is no exact integral for the above differential equation. If we assume that $r_w C_w \ll C_p$ then we can integrate equation (10) to get

$$\theta_e = \theta_d \exp(L_v r_w / T) \qquad (11)$$

where θ_d and θ_e are the potential temperatures for dry air and the equivalent potential temperature respectively.

Some authors have improved the above simplified formula by including adjustment factors (Betts and Dugan 1973, Saunders 1975, Simpson 1978, Bolton 1980). Amongst these formulae, the Bolton's formula

$$\theta_e = T \left(\frac{1000}{p} \right)^{0.2854(1-0.297\times10^{-3} r)}$$

$$\times \exp\left[\left(\frac{3.376}{T_{LCL}} - 0.00254 \right) r \,(1 + 0.81\times10^{-3} r) \right] \qquad (12)$$

best fits the numerically integrated data and agrees to within 0.02 °C.

(i) **Lifting condensation level**

Concept:

The lifting condensation level is defined as the level at which an air parcel lifted adiabatically first becomes saturated.

Method of Deviation:

Referring to figure 1, the lifting condensation level can be found by intersecting any two of the following three curves: (i) the constant mixing ratio line, (ii) the saturation adiabat through the wet-bulb temperature and (iii) the dry adiabat through the air temperature.

Alternatively, it is noted that up to the point when the air parcel is lifted to the lifting condensation level, the mixing ratio and the potential temperature (ignoring the negligible effect of the presence of moisture) of the air parcel are kept constant. This means that up to the point of the lifting condensation level, the vapour pressure is proportional to the total pressure. Then with the use of the Poisson's equation we have

$$\frac{e}{T^{C_p/R}} = \left.\frac{e}{T^{C_p/R}}\right|_{LCL} = \text{constant} = C_{LCL}, \text{ say} \qquad (13)$$

The temperature at the lifting condensation level can be determined from the constant C_{LCL} of the air parcel. The pressure at the lifting condensation level is then obtained by applying the Poisson's equation.

3. COMPARISON AND RECOMMENDATION

(a) Machine used for computation

The computer used for calculation is a 32-bit minicomputer MV 20000 from Data-General and the programs are written in Fortran 77.

(b) Saturation Vapour Pressure over Water

i. Fifteen formulae including the two Goff-Gratch formulae and the Wexler formula were identified. They are listed in Appendix C. Computations were carried out using these formulae for temperature values in the range of meteorological interest. Table 1 shows the comparison with the International Meteorological Tables, Smithsonian Meteorological Tables and their relative speeds of execution.

ii. The Goff-Gratch formula, GG2, agrees exactly with those values tabulated in the International Meteorological Tables where this formula was the generating formula for the tables. The small deviations are due to truncation to 5 significant figures in the International Meteorological Tables.

iii. The Goff-Gratch formula, GG1, does not evaluate to the maximum accuracy for the data tabulated in the Smithsonian Meteorological Tables in spite of the fact that GG1 was stated to be the formula used to generate the tables. This may be attributed to the fact that some of the values contained in the Smithsonian Meteorological Tables were obtained by interpolation.

iv. The Wexler's formula (for temperatures greater than 0°C) agrees with the International and Smithsonian Meteorological Tables to within 0.084 % and 0.071 % respectively for positive Celsius temperatures but deviates to some 1.3 % for negative temperatures.

v. Apart from the Goff-Gratch formula GG2, Sargent's and Wobus' formulae best fit the tabulated data. Both are about three times as fast as the Goff-Gratch formulae and both their mean accuracies are better than 0.02 % and 0.01 % with reference to the International and Smithsonian Meteorological Tables respectively.

vi. Teten's formula TT1, obtained by integrating the Clausius-Clapeyron equation by assuming a constant value of the heat of vaporization and ideal gas law, is accurate to 1.3 % only when compared with the International Meteorological Tables. The improved formula TT2, with the coefficients adjusted, is accurate to only 0.75 %.

Recommendations:

i. The Goff-Gratch formula, GG2, the version adopted for the International Meteorological Tables is to be used as the source of data reference. This replaces the old version GG1 while the Wexler's data have not been universally accepted.

ii. For applications requiring the highest degree of accuracy, the saturation vapour pressure can be obtained by evaluating the Goff-Gratch formula GG2. For other applications, both Wobus' formula and Sargent's formula are very accurate and economical in computation time. These two formulae are recommended for general applications.

iii. Teten's formula is useful for obtaining the inverse function, i.e. temperature as a function of saturation vapour pressure. But as will be shown later in section (3d), there are other formulae which describe this relationship better.

(c) Saturation Vapour Pressure over Ice

i. Three formulae were identified (Appendix D) and compared [Table 2]. Two of them were the Goff-Gratch formulae GG1 (1946) and GG2 (1949) and the third was due to Lowe (1977). The reference data were obtained from the International Meteorological Tables.

ii. The Goff-Gratch formula GG2 agrees with the table to the extent that the precision of the table allows.

iii. The Lowe's formula has a rms percentage deviation of 0.18 % compared to 0.07 % of the Goff-Gratch formula GG2 but the computation speed is triple that of the Goff-Gratch formula.

Recommendation:

i. All three formulae are very accurate. The Goff-Gratch formula GG2 serves as the standard and Lowe's formula, which is computationally the fastest, can be used for practical numerical calculations.

(d) Temperature from Saturation Vapour Pressure

i. Three formulae, one by fitting the Smithsonian Meteorological Tables (Sargent 1980), one by inverting the Simplified Wexler's formula SW1 (Bolton 1980) and one by inverting the Teten's formula were identified (Appendix E) and compared [Table 3]. The reference data were the International Meteorological Tables.

ii. Sargent's formula has a rms percentage deviation of 0.051 % compared to over 0.3 % for Bolton's formula and inverse Teten's formula. Both Sargent's and Bolton's formulae are about 2 times faster to compute than the inverse Teten's formula.

iii. The suggestion (Stipanuk 1973 re: Profs lib) to improve the accuracy of using the inverse Teten's formula by iterative substitution of Teten's formula has also been tested. The results show no obvious improvement in accuracy.

Recommendation:

i. Sargent's formula is both accurate and fast. It is recommended for all applications.

(e) Mixing Ratio

i. In Table 4, saturation mixing ratios were calculated with and without the adjustment coefficient (equations (3) and (3a) in section 2(b)) and compared with the International Meteorological Tables.

ii. Sargent's formula is used for WMR1 and WMR2 for the evaluation of the saturation vapour pressure over water in the above calculations.

iii. The formula for the evaluation of the adjustment coefficient was given by Wobus (Profs Library 1982) and is accurate to within 0.04 % for a pressure greater than 200 hPa, with reference to table 89 of the Smithsonian Meteorological Tables (List 1963).

iv. With the adjustment coefficient included the maximum percentage deviation for the formula WMR2 is 0.094 % compared to 0.479 % for the formula WMR1 without the adjustment coefficient. The relative speed of execution is about 1:1.3.

Recommendation:

i. Since the accuracy is significantly improved by including the adjustment coefficient and the speeds of execution do not differ much, the formula WMR2 (equation (3a)) is recommended for evaluation of the mixing ratio.

(f) Temperature at the Lifting Condensation Level

i. Six formulae were identified (Appendix F: equations (1) to (5) and equation (8)) and compared for the calculation of the temperature at the lifting condensation level.

ii. T_{LCL1} and T_{LCL2} were derived by intersecting the constant mixing ratio line with the dry adiabat. T_{LCL3} was obtained by numerical curve fitting.

iii. T_{LCL4} and T_{LCL5} use equation (13) in section 2(i); the former was obtained by fitting the data tabulated in the Smithsonian Meteorological Tables and the latter was derived from an approximate formula by Bolton (1980).

iv. PTLCL was obtained by approximating the lines of constant mixing ratio lines and the dry adiabats on the skew T - log P chart by straight lines (Profs lib 1980). The temperature and pressure at the lifting condensation level are both calculated in this subroutine.

v. The reference data TLCLFIT were obtained by solving equation (13) of section 2(i) numerically. The Goff-Gratch formula GG2 was used for the calculation of the saturation vapour pressure. The results of comparison are given in Table 5.

vi. For all the six formulae, the maximum deviation from the reference data is rather small (less than 0.5°C). Among them, T_{LCL2} deviates least (< 0.05 °C) but takes twice the time for T_{LCL3}, which has a maximum deviation barely exceeding 0.1 °C.

vii. Other formulae are less accurate and slower than both T_{LCL2} and T_{LCL3}.

viii. The computation of the reference data, TLCLFIT, requires a reasonably good first guess and the speed of computation depends on the precision required. In general the speed is about 200 times slower T_{LCL2}.

Recommendations:

i. T_{LCL3} is computationally the fastest and is reasonably accurate (< 0.1°C), and can be used for calculations when the temperature at the lifting condensation level has to be evaluated many times and the speed of computation is a major consideration.

ii. For applications requiring higher precision, the algorithm T_{LCL2} should be used.

(g) Pressure at the Lifting Condensation Level

i. Pressure at the lifting condensation level is calculated from the computed temperature (see the above section) at the lifting condensation level and the given initial pressure and temperature of the air parcel by using Poisson's equation. PTLCL evaluates both temperature and pressure at the lifting condensation level in the same subroutine (section (3f)).

ii. The reference data PFIT1 was computed from the temperature reference TLCLFIT using Poisson's equation with moisture taken into account while PFIT2 was computed in a similar manner but the small effect of moisture in the dry adiabatic process was ignored. The results of comparison are given in Table 6.

iii. There is no significant difference between the pressures obtained by PFIT1 and PFIT2 with and without including the mixing ratio in Poisson's equation. This leads support to the argument given in section (2g). However, the latter is about twice as fast as the former.

iv. The difference in the pressures PFIT1 and PLCL2 (obtained by using TLCL2 as the temperature at the lifting condensation level) is small and is less than 0.4 hPa.

v. The use of TLCL3 instead of TLCL2 to give the pressure PLCL3 is reasonably accurate although the maximum deviation is up to 0.8 hPa.

vi. The pressure computed using PTLCL shows a large deviation especially when the initial pressure of the air parcel is high.

Recommendations:

i. PLCL2 which is derived from the temperature TLCL2 at the lifting condensation level using the dry adiabatic approximation is recommended for the calculation of the pressure at the lifting condensation level.

ii. PLCL3 is only marginally inferior to PLCL2. Considering that computing TLCL3 takes less time, PLCL3 is preferred if computation speed is a major concern.

(h) Equivalent Potential Temperature, Wet-Bulb Potential Temperature and Saturation Adiabat

i. Two formulae for the conversion between the equivalent potential temperature (EPT) and the wet-bulb potential temperature (WPT) were obtained by fitting to the values tabulated in the Smithsonian Meteorological Tables. The maximum deviations are 0.185°C for the 'WPT → EPT' conversion and 0.056°C for the 'EPT → WPT' conversion [Tables 7 & 8].

ii. For equivalent potential temperature, values obtained from the simplified formula PTEQ1 (equation (1) in Appendix G) and the improved formula PTEQ2 (equation (2) in Appendix G) were compared. The maximum deviation between the two formulae is less than 1.5°C but the simplified formula is faster than the other by about 1.5 times [Table 9].

iii. Three algorithms for the calculation of the wet-bulb potential temperature were identified and compared. Both of the two algorithms WPT1 and WPT2 make use of the corresponding equivalent potential temperature, evaluated by PTEQ2. While WPT1 is the numerically fitted formula in paragraph 1 in this section, WPT2 evaluates the wet-bulb potential temperature by finding the wet-bulb temperature at the 1000 hPa level, on a saturation adiabat defined by the equivalent potential temperature. This is achieved by iterative substitution of PTEQ2. WPT3 is a separate empirical formula. The results of comparison are shown in Table 10.

iv. There is no reference data available for the direct evaluation of wet-bulb potential temperature. We may use WPT1 which is correct to within 1°C (error in PTEQ2 + error in WPT1) as the reference data. The maximum deviation of WPT2 from WPT1 is up to 0.26 °C. For WPT3 the deviation is larger for higher dry-bulb temperatures at each pressure level, and exceeds 1.5 °C for levels higher than 700 hPa [Table 10].

v. The relative time of execution for obtaining WPT1, WPT2 and WPT3 is about 1 : 16.3 : 1.2.

vi. To evaluate the temperature on a saturation adiabat given the equivalent potential temperature and pressure or alternatively to find the pressure on a saturation adiabat given the equivalent potential temperature and temperature, a convenient method is iterative substitution of the formula for the equivalent potential temperature (for example items (7) (8) and (9) in Appendix G) to the required precision. The accuracy and speed of computation depend on the choice of the iteration formula and the convergence scheme.

Recommendations:

i. For the evaluation of the equivalent potential temperature for a saturation adiabat, Bolton's formula is recommended. The simplified formula ((1) in Appendix G) may be used for high speed computation and when the accuracy requirement is more relaxed.

ii. For the calculation of the wet-bulb potential temperature, WPT1 is recommended because WPT2 is slower and WPT3 gives large deviations. We also note that the function f in WPT3 (equation (6) in Appendix G) may approach zero, in which case the evaluation of the WPT3 becomes impossible.

(i) Other Parameters:

i. Equation 4 in appendix H evaluates the temperature, given the saturation mixing ratio and pressure of the air parcel, and equation 5 in the same appendix gives the dew point from the relative humidity and temperature. They are accurate to 0.24 °C and 0.16 °C respectively.

ii. For other parameters where the methods of evaluation have not been mentioned in previous discussions, they can be obtained by a combination of the formulae and/or algorithms listed in the appendices.

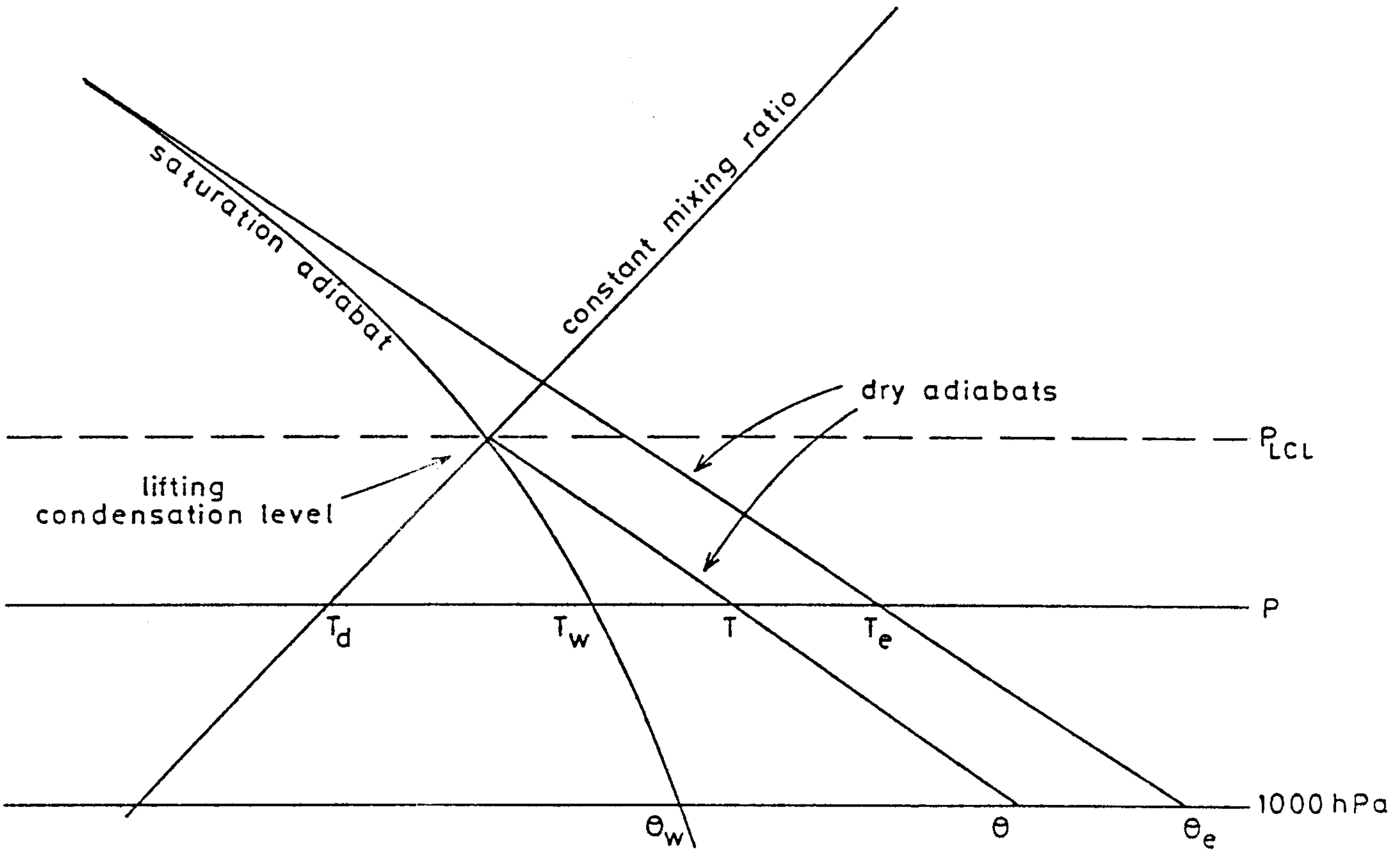

Figure 1 Saturated and dry adiabats on a tephigram.

TABLE 1. SATURATION VAPOUR PRESSURE OVER WATER

		International Meteorological Tables				Smithsonian Meteorological Tables				Time
		a1	a2	b1	b2	a1	a2	b1	b2	
1	GG1	.0626	.0610	.123	.090	.0780	.0756	.175	.112	2.9
2	GG2	.0061	.0030	.040	.015	.0183	.0155	.092	.040	3.0
3	WEX	.3899	.1906	1.425	.563	.3772	.1775	1.393	.539	1.3
4	SG	.0192	.0159	.067	.038	.0098	.0037	.096	.027	1.1
5	SW1	.2217	.1881	.542	.337	.2098	.1750	.542	.323	0.8
6	SW2	.3910	.2245	1.310	.411	.3899	.2134	1.341	.393	1.4
7	TT1	1.2872	.6281	4.322	2.026	1.2987	.6383	4.352	2.026	1.2
8	TT2	.7471	.3278	2.624	1.133	.7584	.3379	2.655	1.133	1.2
9	WOB	.0171	.0148	.061	.037	.0090	.0041	.090	.026	1.0
10	NQ	.0657	.0642	.126	.093	.0811	.0788	.177	.117	2.8
11	RC	.0405	.0387	.106	.104	.0528	.0502	.144	.127	1.0
12	LW	.1403	.0729	.587	.232	.1385	.0797	.610	.257	0.8
13	3B	1.0009	1.0342	1.344	1.344	.9907	1.0225	1.337	1.337	1.4
14	TB1	5.9173	3.1808	18.97	9.499	5.9043	3.1711	18.93	9.499	1.2
15	TB2	.6682	.4260	1.879	1.136	.6563	.4173	1.847	1.114	1.3

a - temperature range from -49.9 to 50.9 deg C in steps of 0.1 deg C
b - temperature range from -35.9 to 25.9 deg C in steps of 0.1 deg C
1 - rms percentage deviation
2 - maximum percentage deviation
Time - relative time of execution

TABLE 2. SATURATION VAPOUR PRESSURE OVER ICE

TEMP	IMT	GG2 (PCDV)	GG1 (PCDV)	LW (PCDV)
-42.0	.1021	.10210(.0026)	.10211(.0142)	.10212(.0153)
-41.0	.1145	.11450(.0043)	.11452(.0160)	.11453(.0242)
-40.0	.1283	.12829(-.0103)	.12830(.0022)	.12832(.0124)
-39.0	.1436	.14359(-.0097)	.14360(.0029)	.14360(-.0017)
-38.0	.1606	.16056(-.0260)	.16058(-.0128)	.16054(-.0358)
-37.0	.1794	.17937(-.0161)	.17939(-.0045)	.17933(-.0370)
-36.0	.2002	.20020(-.0024)	.20022(.0094)	.20014(-.0286)
-35.0	.2232	.22324(.0160)	.22326(.0284)	.22316(-.0202)
-34.0	.2487	.24870(-.0004)	.24873(.0120)	.24861(-.0375)
-33.0	.2768	.27682(.0058)	.27685(.0185)	.27672(-.0285)
-32.0	.3078	.30784(.0133)	.30788(.0266)	.30775(-.0176)
-31.0	.3420	.34204(.0121)	.34208(.0239)	.34196(-.0128)
-30.0	.3797	.37972(.0041)	.37976(.0161)	.37965(-.0141)
-29.0	.4212	.42118(-.0056)	.42123(.0069)	.42113(-.0166)
-28.0	.4668	.46676(-.0076)	.46682(.0051)	.46676(-.0088)
-27.0	.5169	.51686(-.0072)	.51692(.0037)	.51689(-.0020)
-26.0	.5719	.57186(-.0072)	.57194(.0064)	.57193(.0049)
-25.0	.6322	.63219(-.0013)	.63228(.0124)	.63230(.0163)
-24.0	.6983	.69833(.0041)	.69841(.0163)	.69848(.0252)
-23.0	.7708	.77077(-.0035)	.77087(.0093)	.77094(.0188)
-22.0	.8501	.85006(-.0048)	.85017(.0079)	.85026(.0182)
-21.0	.9368	.93677(-.0032)	.93689(.0100)	.93698(.0188)
-20.0	1.0315	1.03153(.0025)	1.03167(.0161)	1.03174(.0232)
-19.0	1.1350	1.13501(.0008)	1.13514(.0125)	1.13521(.0184)
-18.0	1.2479	1.24794(.0029)	1.24808(.0147)	1.24811(.0167)
-17.0	1.3711	1.37109(-.0010)	1.37126(.0115)	1.37121(.0081)
-16.0	1.5053	1.50526(-.0027)	1.50545(.0100)	1.50535(.0032)
-15.0	1.6514	1.65138(-.0010)	1.65160(.0121)	1.65141(.0007)
-14.0	1.8104	1.81040(.0002)	1.81060(.0113)	1.81035(-.0026)
-13.0	1.9833	1.98327(-.0014)	1.98350(.0103)	1.98320(-.0052)
-12.0	2.1712	2.17116(-.0018)	2.17142(.0099)	2.17104(-.0075)
-11.0	2.3752	2.37519(-.0003)	2.37549(.0120)	2.37504(-.0066)
-10.0	2.5966	2.59662(.0006)	2.59694(.0131)	2.59646(-.0053)
-9.0	2.8368	2.83677(-.0009)	2.83707(.0095)	2.83663(-.0060)
-8.0	3.0970	3.09699(-.0003)	3.09740(.0130)	3.09696(-.0011)
-7.0	3.3789	3.37889(-.0003)	3.37928(.0111)	3.37898(.0025)
-6.0	3.6840	3.68399(-.0003)	3.68450(.0135)	3.68430(.0081)
-5.0	4.0141	4.01411(.0002)	4.01459(.0121)	4.01462(.0130)
-4.0	4.3709	4.37088(-.0005)	4.37140(.0115)	4.37177(.0200)
-3.0	4.7564	4.75646(.0013)	4.75706(.0140)	4.75769(.0272)
-2.0	5.1727	5.17271(.0003)	5.17338(.0131)	5.17443(.0334)
-1.0	5.6219	5.62184(-.0010)	5.62259(.0122)	5.62416(.0401)
.0	6.1064	6.10640(.0000)	6.10710(.0115)	6.10918(.0455)
Rms percentage deviation		.007	.013	.018
Max percentage deviation		.026	.028	.045
Relative time of execution		3	5	1

PCDV - Percentage deviation

TABLE 3. TEMPERATURE FROM SATURATION VAPOUR PRESSURE OVER WATER

SVP	IMT	SG (PCDV)	BT (PCDV)	ITT (PCDV)
.06354	-50.0	-49.998(-.004)	-50.005(.009)	-49.619(-.762)
.08432	-47.5	-47.500(.001)	-47.511(.023)	-47.155(-.727)
.1111	-45.0	-45.003(.006)	-45.017(.038)	-44.690(-.689)
.1454	-42.5	-42.504(.009)	-42.523(.053)	-42.223(-.652)
.1891	-40.0	-40.002(.005)	-40.025(.062)	-39.752(-.620)
.2443	-37.5	-37.504(.010)	-37.530(.080)	-37.282(-.581)
.3138	-35.0	-35.002(.007)	-35.032(.090)	-34.808(-.549)
.4007	-32.5	-32.501(.004)	-32.533(.101)	-32.332(-.517)
.5087	-30.0	-30.002(.007)	-30.035(.116)	-29.856(-.480)
.6423	-27.5	-27.503(.010)	-27.536(.132)	-27.378(-.444)
.8068	-25.0	-25.002(.009)	-25.036(.144)	-24.897(-.412)
1.0082	-22.5	-22.503(.012)	-22.536(.159)	-22.415(-.378)
1.2538	-20.0	-20.002(.010)	-20.034(.169)	-19.930(-.349)
1.5516	-17.5	-17.503(.015)	-17.533(.186)	-17.445(-.315)
1.9114	-15.0	-15.002(.016)	-15.030(.202)	-14.957(-.285)
2.3441	-12.5	-12.502(.018)	-12.527(.219)	-12.468(-.256)
2.8622	-10.0	-10.002(.023)	-10.025(.246)	-9.977(-.225)
3.4802	-7.5	-7.502(.026)	-7.521(.283)	-7.485(-.197)
4.2142	-5.0	-5.002(.042)	-5.018(.363)	-4.992(-.157)
5.0829	-2.5	-2.502(.076)	-2.515(.588)	-2.497(-.102)
5.4726	-1.5	-1.502(.131)	-1.513(.898)	-1.499(-.039)
5.6772	-1.0	-1.002(.204)	-1.013(1.291)	-1.000(.035)
5.8886	-.5	-.502(.400)	-.512(2.449)	-.501(.230)
6.3324	.5	.498(-.406)	.489(-2.204)	.497(-.546)
6.5653	1.0	.998(-.188)	.990(-1.024)	.997(-.329)
6.8057	1.5	1.498(-.123)	1.490(-.640)	1.496(-.262)
7.0538	2.0	1.998(-.097)	1.991(-.455)	1.995(-.233)
7.3099	2.5	2.498(-.079)	2.491(-.343)	2.495(-.211)
7.5743	3.0	2.998(-.061)	2.992(-.261)	2.994(-.190)
8.1284	4.0	3.998(-.048)	3.993(-.169)	3.993(-.169)
8.7181	5.0	4.998(-.038)	4.994(-.113)	4.992(-.152)
9.3453	6.0	5.998(-.031)	5.995(-.076)	5.992(-.138)
10.012	7.0	6.998(-.026)	6.996(-.052)	6.991(-.126)
10.720	8.0	7.998(-.029)	7.997(-.039)	7.990(-.122)
11.473	9.0	8.999(-.013)	8.999(-.014)	8.991(-.100)
12.271	10.0	9.998(-.016)	9.999(-.009)	9.990(-.096)
13.118	11.0	10.999(-.013)	11.000(.000)	10.990(-.087)
14.016	12.0	11.999(-.011)	12.001(.005)	11.991(-.079)
14.967	13.0	12.998(-.014)	13.001(.005)	12.990(-.076)
15.975	14.0	13.998(-.012)	14.001(.008)	13.990(-.069)
17.042	15.0	14.998(-.011)	15.001(.009)	14.991(-.062)
18.171	16.0	15.998(-.011)	16.002(.009)	15.991(-.057)
19.365	17.0	16.998(-.011)	17.001(.008)	16.991(-.052)
20.628	18.0	17.999(-.008)	18.002(.010)	17.992(-.044)
21.962	19.0	18.999(-.008)	19.002(.008)	18.993(-.039)
23.371	20.0	19.999(-.007)	20.001(.006)	19.993(-.034)
24.858	21.0	20.998(-.008)	21.000(.002)	20.993(-.031)
26.428	22.0	21.999(-.007)	22.000(.000)	21.994(-.026)

TABLE 3. (cont'd)

SVP	IMT	SG (PCDV)	BT (PCDV)	TT (PCDV)
28.083	23.0	22.998(-.008)	22.999(-.005)	22.995(-.023)
29.829	24.0	23.999(-.006)	23.998(-.008)	23.996(-.017)
31.668	25.0	24.998(-.006)	24.997(-.013)	24.996(-.015)
33.606	26.0	25.999(-.005)	25.996(-.016)	25.997(-.010)
35.646	27.0	26.999(-.005)	26.994(-.021)	26.998(-.007)
37.793	28.0	27.999(-.005)	27.993(-.027)	27.999(-.004)
40.052	29.0	28.999(-.005)	28.991(-.032)	29.000(-.001)
42.427	30.0	29.999(-.005)	29.988(-.038)	30.000(.001)
44.924	31.0	30.999(-.005)	30.986(-.044)	31.001(.003)
47.548	32.0	31.999(-.004)	31.984(-.050)	32.002(.006)
50.303	33.0	32.999(-.004)	32.981(-.057)	33.002(.007)
53.197	34.0	33.999(-.003)	33.978(-.063)	34.003(.010)
56.233	35.0	34.999(-.003)	34.975(-.071)	35.004(.012)
59.418	36.0	35.999(-.004)	35.972(-.078)	36.005(.013)
62.759	37.0	36.999(-.003)	36.968(-.086)	37.005(.014)
66.260	38.0	37.999(-.003)	37.964(-.094)	38.006(.015)
69.930	39.0	38.999(-.003)	38.960(-.101)	39.006(.016)
73.773	40.0	39.999(-.003)	39.956(-.110)	40.007(.016)
77.798	41.0	40.999(-.003)	40.951(-.118)	41.007(.017)
82.011	42.0	41.999(-.002)	41.947(-.127)	42.007(.018)
86.419	43.0	42.999(-.002)	42.942(-.136)	43.008(.018)
91.029	44.0	43.999(-.002)	43.936(-.145)	44.008(.017)
95.850	45.0	44.999(-.002)	44.930(-.154)	45.008(.017)
100.89	46.0	45.999(-.002)	45.925(-.163)	46.008(.017)
106.15	47.0	46.998(-.004)	46.918(-.175)	47.007(.014)
111.65	48.0	47.998(-.003)	47.911(-.185)	48.007(.014)
117.40	49.0	49.000(-.001)	48.906(-.192)	49.008(.016)
123.39	50.0	49.999(-.002)	49.898(-.204)	50.006(.013)
126.48	50.5	50.498(-.003)	50.394(-.210)	50.506(.011)
Rms percentage deviation		.051	.316	.335
Max percentage deviation		.406	2.449	.762
Relative time of execution		1	1	2

IMT - International Meteorological Tables
SG - Sargent's formula
BT - Bolton's formula
ITT - Inverse Teten's formulae
PCDV - Percentage deviation
Temperature range for calculating deviations:
-50 to 50.5 deg C in steps of 0.5 deg C

TABLE 4. SATURATION MIXING RATIO

PRES	TEMP	IMT	WMR1	PCDV	WMR2	PCDV
1000	-5.0	2.6440	2.6326	(-.429)	2.6447	(.026)
1000	.0	3.8380	3.8223	(-.410)	3.8396	(.043)
1000	5.0	5.4940	5.4709	(-.420)	5.4957	(.032)
1000	10.0	7.7610	7.7280	(-.425)	7.7633	(.030)
1000	15.0	10.8310	10.7848	(-.427)	10.8348	(.035)
1000	20.0	14.9510	14.8855	(-.438)	14.9560	(.033)
1000	25.0	20.4370	20.3429	(-.460)	20.4422	(.025)
1000	30.0	27.6930	27.5602	(-.479)	27.6998	(.025)
850	-10.0	2.1100	2.1018	(-.387)	2.1101	(.006)
850	-5.0	3.1110	3.0995	(-.368)	3.1116	(.019)
850	.0	4.5180	4.5017	(-.361)	4.5191	(.023)
850	5.0	6.4700	6.4464	(-.365)	6.4713	(.020)
850	10.0	9.1460	9.1117	(-.375)	9.1473	(.014)
850	15.0	12.7760	12.7269	(-.384)	12.7775	(.012)
850	20.0	17.6550	17.5866	(-.387)	17.6585	(.020)
700	-15.0	1.7090	1.7034	(-.331)	1.7090	(.000)
700	-10.0	2.5620	2.5541	(-.309)	2.5624	(.014)
700	-5.0	3.7790	3.7678	(-.297)	3.7798	(.021)
700	.0	5.4920	5.4748	(-.313)	5.4922	(.004)
700	5.0	7.8700	7.8451	(-.316)	7.8702	(.003)
700	10.0	11.1340	11.0991	(-.313)	11.1351	(.010)
500	-30.0	.6351	.6336	(-.241)	.6353	(.026)
500	-25.0	1.0078	1.0055	(-.232)	1.0080	(.021)
500	-20.0	1.5670	1.5639	(-.200)	1.5677	(.042)
500	-15.0	2.3930	2.3873	(-.238)	2.3929	(-.004)
500	-10.0	3.5900	3.5816	(-.234)	3.5898	(-.005)
500	-5.0	5.3000	5.2877	(-.232)	5.2997	(-.006)
300	-40.0	.3930	.3924	(-.156)	.3931	(.022)
300	-35.0	.6523	.6514	(-.136)	.6525	(.028)
300	-30.0	1.0581	1.0567	(-.136)	1.0583	(.017)
300	-25.0	1.6798	1.6776	(-.132)	1.6800	(.012)
300	-20.0	2.6140	2.6108	(-.122)	2.6144	(.017)
200	-50.0	.1979	.1978	(-.058)	.1980	(.067)
200	-45.0	.3461	.3459	(-.069)	.3463	(.044)
200	-40.0	.5893	.5888	(-.090)	.5894	(.012)
200	-35.0	.9785	.9776	(-.089)	.9786	(.007)
200	-30.0	1.5877	1.5863	(-.085)	1.5878	(.006)
Max percentage deviation				.479		.094
Relative time of execution				1		1.3

IMT - International Meteorological Tables (WMO 1966)
WMR1 - Without including deviation coefficient from ideal gas
WMR2 - Coefficient for deviation from ideal gas included
PCDV - Percentage deviation

TABLE 5. TEMPERATURE AT THE LIFTING CONDENSATION LEVEL

TEMP	DEWPT	TLCLFIT	TLCL1	TLCL2	TLCL3	TLCL4	TLCL5	PTLCL
-30.0	-50.0	-52.986	-52.857	-52.945	-52.883	-53.336	-52.652	-52.814
-30.0	-45.0	-47.350	-47.218	-47.327	-47.284	-47.441	-47.087	-47.242
-30.0	-40.0	-41.642	-41.622	-41.632	-41.610	-41.645	-41.430	-41.586
-30.0	-35.0	-35.860	-35.911	-35.858	-35.851	-35.832	-35.681	-35.840
-30.0	-30.0	-30.000	-30.078	-30.000	-30.000	-29.984	-29.837	-30.000
-25.0	-45.0	-48.089	-47.948	-48.058	-48.001	-48.203	-47.818	-47.920
-25.0	-40.0	-42.428	-42.400	-42.413	-42.374	-42.434	-42.210	-42.323
-25.0	-35.0	-36.695	-36.741	-36.690	-36.671	-36.668	-36.513	-36.642
-25.0	-30.0	-30.887	-30.961	-30.887	-30.883	-30.879	-30.723	-30.869
-25.0	-25.0	-25.000	-25.078	-25.000	-25.000	-24.988	-24.839	-25.000
-20.0	-40.0	-43.192	-43.157	-43.171	-43.118	-43.215	-42.968	-43.027
-20.0	-35.0	-37.507	-37.546	-37.499	-37.464	-37.477	-37.320	-37.406
-20.0	-30.0	-31.748	-31.819	-31.748	-31.733	-31.730	-31.583	-31.698
-20.0	-25.0	-25.914	-25.990	-25.916	-25.914	-25.898	-25.753	-25.898
-20.0	-20.0	-20.000	-20.071	-20.000	-20.000	-19.992	-19.830	-20.000
-15.0	-35.0	-38.296	-38.329	-38.284	-38.236	-38.273	-38.105	-38.136
-15.0	-30.0	-32.586	-32.652	-32.585	-32.554	-32.570	-32.418	-32.489
-15.0	-25.0	-26.802	-26.876	-26.806	-26.794	-26.797	-26.642	-26.755
-15.0	-20.0	-20.942	-21.012	-20.946	-20.945	-20.941	-20.774	-20.927
-15.0	-15.0	-15.000	-15.062	-15.000	-15.000	-14.992	-14.814	-15.000
-10.0	-30.0	-33.400	-33.462	-33.398	-33.353	-33.371	-33.230	-33.246
-10.0	-25.0	-27.666	-27.737	-27.671	-27.644	-27.648	-27.505	-27.573
-10.0	-20.0	-21.857	-21.927	-21.864	-21.855	-21.852	-21.692	-21.812
-10.0	-15.0	-15.970	-16.033	-15.975	-15.977	-15.969	-15.788	-15.956
-10.0	-10.0	-10.000	-10.053	-10.000	-10.000	-10.000	-9.792	-10.000
-5.0	-25.0	-28.506	-28.574	-28.512	-28.470	-28.484	-28.345	-28.356
-5.0	-20.0	-22.747	-22.816	-22.758	-22.734	-22.734	-22.583	-22.659
-5.0	-15.0	-16.912	-16.976	-16.923	-16.916	-16.922	-16.734	-16.870
-5.0	-10.0	-10.998	-11.052	-11.005	-11.008	-10.984	-10.795	-10.986
-5.0	-5.0	-5.000	-5.043	-5.000	-5.000	-5.004	-4.766	-5.000
.0	-20.0	-23.613	-23.681	-23.626	-23.588	-23.594	-23.450	-23.469
.0	-15.0	-17.829	-17.893	-17.845	-17.824	-17.820	-17.654	-17.745
.0	-10.0	-11.968	-12.024	-11.981	-11.978	-11.961	-11.770	-11.929
.0	-5.0	-6.027	-6.072	-6.034	-6.039	-6.027	-5.798	-6.016
.0	.0	.000	-.032	.000	.000	.008	.262	.000
5.0	-25.0	-30.120	-30.182	-30.127	-30.069	-30.094	-29.957	-29.825
5.0	-20.0	-24.456	-24.522	-24.472	-24.421	-24.438	-24.294	-24.245
5.0	-15.0	-18.721	-18.786	-18.741	-18.705	-18.715	-18.549	-18.582
5.0	-10.0	-12.912	-12.970	-12.931	-12.914	-12.906	-12.718	-12.832
5.0	-5.0	-7.025	-7.073	-7.040	-7.039	-7.023	-6.802	-6.989
5.0	.0	-1.056	-1.091	-1.064	-1.071	-1.047	-.800	-1.047
5.0	5.0	5.000	4.978	5.000	5.000	5.008	5.290	5.000
10.0	-20.0	-25.277	-25.342	-25.295	-25.237	-25.250	-25.116	-24.989
10.0	-15.0	-19.590	-19.654	-19.614	-19.564	-19.590	-19.420	-19.385
10.0	-10.0	-13.831	-13.890	-13.856	-13.823	-13.820	-13.641	-13.697
10.0	-5.0	-7.997	-8.046	-8.019	-8.004	-7.980	-7.779	-7.920
10.0	.0	-2.083	-2.120	-2.099	-2.100	-2.070	-1.833	-2.049
10.0	5.0	3.914	3.890	3.906	3.898	3.922	4.198	3.923
10.0	10.0	10.000	9.987	10.000	10.000	10.004	10.315	10.000

TABLE 5. (cont'd)

TEMP	DEWPT	TLCLFIT	TLCL1	TLCL2	TLCL3	TLCL4	TLCL5	PTLCL
15.0	-15.0	-20.437	-20.501	-20.464	-20.406	-20.422	-20.269	-20.155
15.0	-10.0	-14.726	-14.786	-14.756	-14.708	-14.719	-14.540	-14.526
15.0	-5.0	-8.943	-8.995	-8.971	-8.940	-8.930	-8.730	-8.813
15.0	.0	-3.083	-3.123	-3.106	-3.094	-3.066	-2.839	-3.009
15.0	5.0	2.858	2.832	2.842	2.838	2.867	3.136	2.891
15.0	10.0	8.884	8.869	8.877	8.867	8.895	9.194	8.892
15.0	15.0	15.000	14.990	15.000	15.000	15.016	15.337	15.000
20.0	-10.0	-15.598	-15.659	-15.633	-15.574	-15.590	-15.416	-15.323
20.0	-5.0	-9.864	-9.918	-9.899	-9.851	-9.852	-9.657	-9.670
20.0	.0	-4.056	-4.098	-4.087	-4.057	-4.047	-3.818	-3.930
20.0	5.0	1.829	1.801	1.806	1.816	1.848	2.102	1.901
20.0	10.0	7.798	7.781	7.783	7.777	7.812	8.103	7.829
20.0	15.0	13.853	13.844	13.847	13.836	13.855	14.185	13.860
20.0	20.0	20.000	19.987	20.000	20.000	20.012	20.351	20.000
25.0	-5.0	-10.763	-10.818	-10.803	-10.742	-10.758	-10.559	-10.492
25.0	.0	-5.005	-5.049	-5.042	-4.995	-4.992	-4.772	-4.814
25.0	5.0	.828	.797	.798	.825	.844	1.094	.951
25.0	10.0	6.741	6.723	6.719	6.726	6.758	7.040	6.810
25.0	15.0	12.737	12.727	12.724	12.716	12.750	13.065	12.767
25.0	20.0	18.822	18.811	18.817	18.804	18.840	19.170	18.828
25.0	25.0	25.000	24.975	25.000	25.000	25.008	25.357	25.000
30.0	.0	-5.930	-5.976	-5.973	-5.911	-5.910	-5.702	-5.664
30.0	5.0	-.148	-.181	-.185	-.138	-.141	.113	.039
30.0	10.0	5.711	5.691	5.682	5.708	5.730	6.004	5.831
30.0	15.0	11.650	11.640	11.631	11.636	11.668	11.973	11.718
30.0	20.0	17.675	17.666	17.665	17.654	17.688	18.020	17.704
30.0	25.0	23.790	23.770	23.787	23.773	23.797	24.146	23.796
30.0	30.0	30.000	29.951	30.000	30.000	30.012	30.352	30.000
35.0	5.0	-1.099	-1.134	-1.144	-1.079	-1.090	-.844	-.838
35.0	10.0	4.707	4.686	4.671	4.718	4.727	4.995	4.891
35.0	15.0	10.592	10.580	10.566	10.590	10.602	10.909	10.710
35.0	20.0	16.558	16.550	16.543	16.546	16.563	16.900	16.625
35.0	25.0	22.612	22.595	22.606	22.593	22.625	22.967	22.641
35.0	30.0	28.758	28.716	28.757	28.742	28.770	29.111	28.764
35.0	35.0	35.000	34.913	35.000	35.000	34.996	35.334	35.000
Max deviation from TLCLFIT								
			.141	.044	.103	.349	.357	.357
Relative time of execution								
		180	5	2	1	3	5	5

TABLE 6. PRESSURE AT THE LIFTING CONDENSATION LEVEL

PRES	TEMP	DEWPT	TLCLFIT	PFIT1	PFIT2	PLCL2	PLCL3	PTLCL
1000	-5.0	-20.0	-22.75	786.831	786.834	786.719	786.973	793.95
1000	-5.0	-10.0	-11.00	923.849	923.853	923.771	923.732	924.76
1000	.0	-15.0	-17.83	789.522	789.527	789.358	789.576	796.43
1000	.0	-5.0	-6.03	924.857	924.862	924.770	924.708	925.73
1000	5.0	-10.0	-12.91	792.105	792.112	791.910	792.092	798.82
1000	5.0	.0	-1.06	925.822	925.829	925.731	925.652	926.66
1000	10.0	.0	-2.08	858.385	858.396	858.223	858.207	861.49
1000	10.0	5.0	3.91	926.740	926.750	926.658	926.563	927.56
1000	15.0	5.0	2.86	860.062	860.079	859.911	859.868	863.08
1000	15.0	10.0	8.88	927.630	927.644	927.556	927.444	928.42
1000	20.0	5.0	1.83	799.272	799.294	799.058	799.152	805.48
1000	20.0	15.0	13.85	928.472	928.491	928.422	928.295	929.24
1000	25.0	10.0	6.74	801.475	801.505	801.287	801.356	807.54
1000	25.0	20.0	18.82	929.285	929.311	929.257	929.115	930.04
1000	30.0	15.0	11.65	803.594	803.636	803.445	803.494	809.53
1000	30.0	25.0	23.79	930.063	930.099	930.064	929.912	930.81
850	-10.0	-20.0	-21.86	723.295	723.298	723.221	723.315	726.30
850	-10.0	-15.0	-15.97	784.385	784.388	784.325	784.310	785.19
850	-5.0	-20.0	-22.75	668.805	668.809	668.711	668.927	674.86
850	-5.0	-15.0	-16.91	724.979	724.983	724.875	724.937	727.88
850	-5.0	-10.0	-11.00	785.271	785.275	785.206	785.172	786.05
850	.0	-15.0	-17.83	671.093	671.098	670.954	671.140	676.97
850	.0	-10.0	-11.97	726.590	726.596	726.468	726.503	729.40
850	.0	-5.0	-6.03	786.128	786.133	786.054	786.002	786.87
850	5.0	-10.0	-12.91	673.288	673.296	673.123	673.278	679.00
850	5.0	-5.0	-7.03	728.140	728.149	728.006	728.015	730.86
850	5.0	.0	-1.06	786.948	786.955	786.871	786.804	787.66
850	10.0	-5.0	-8.00	675.398	675.409	675.218	675.344	680.95
850	10.0	.0	-2.08	729.625	729.637	729.490	729.476	732.27
850	10.0	5.0	3.91	787.727	787.737	787.659	787.579	788.42
850	15.0	.0	-3.08	677.430	677.446	677.242	677.345	682.84
850	15.0	5.0	2.86	731.050	731.067	730.924	730.888	733.62
850	15.0	10.0	8.88	788.483	788.497	788.423	788.327	789.15
850	20.0	5.0	1.83	679.378	679.400	679.199	679.279	684.66
850	20.0	15.0	13.85	789.198	789.218	789.159	789.051	789.86
700	-15.0	-30.0	-32.59	546.788	546.789	546.799	547.040	552.09
700	-15.0	-20.0	-20.94	645.195	645.197	645.161	645.165	645.90
700	-10.0	-25.0	-27.67	548.823	548.825	548.786	548.995	553.96
700	-10.0	-15.0	-15.97	645.964	645.966	645.915	645.902	646.63
700	-5.0	-20.0	-22.75	550.780	550.784	550.703	550.881	555.77
700	-5.0	-10.0	-11.00	646.694	646.697	646.640	646.613	647.33
700	.0	-15.0	-17.83	552.664	552.669	552.551	552.703	557.50
700	.0	-5.0	-6.03	647.398	647.403	647.339	647.295	648.01
700	5.0	-10.0	-12.91	554.471	554.479	554.337	554.464	559.17
700	5.0	.0	-1.06	648.073	648.081	648.011	647.956	648.66
700	10.0	-5.0	-8.00	556.209	556.219	556.062	556.166	560.78
700	10.0	5.0	3.91	648.715	648.725	648.661	648.594	649.29

TABLE 6. (cont'd)

PRES	TEMP	DEWPT	TLCLFIT	PFIT1	PFIT2	PLCL2	PLCL3	PTLCL
500	-35.0	-50.0	-52.27	384.136	384.136	384.327	384.614	388.43
500	-35.0	-40.0	-40.83	458.418	458.418	458.452	458.510	459.02
500	-30.0	-45.0	-47.35	385.838	385.838	385.979	386.234	390.00
500	-30.0	-35.0	-35.86	459.067	459.067	459.083	459.125	459.64
500	-25.0	-40.0	-42.43	387.476	387.477	387.568	387.793	391.51
500	-25.0	-30.0	-30.89	459.688	459.689	459.688	459.717	460.24
500	-20.0	-35.0	-37.51	389.050	389.051	389.098	389.295	392.96
500	-20.0	-25.0	-25.91	460.283	460.284	460.270	460.285	460.81
500	-15.0	-30.0	-32.59	390.562	390.564	390.571	390.743	394.35
500	-15.0	-20.0	-20.94	460.853	460.855	460.830	460.832	461.35
500	-10.0	-25.0	-27.67	392.016	392.018	391.990	392.139	395.69
500	-10.0	-15.0	-15.97	461.402	461.405	461.368	461.359	461.88
500	-5.0	-20.0	-22.75	393.414	393.417	393.359	393.487	396.98
500	-5.0	-10.0	-11.00	461.923	461.926	461.885	461.866	462.38
500	.0	-15.0	-17.83	394.759	394.763	394.679	394.788	398.22
300	-50.0	-65.0	-67.04	227.174	227.174	227.368	227.611	229.99
300	-50.0	-55.0	-55.75	273.778	273.778	273.834	273.902	274.19
300	-45.0	-60.0	-62.12	228.318	228.318	228.489	228.706	231.06
300	-45.0	-50.0	-50.78	274.220	274.220	274.265	274.320	274.62
300	-40.0	-55.0	-57.19	229.421	229.421	229.564	229.758	232.08
300	-40.0	-45.0	-45.81	274.644	274.644	274.677	274.721	275.02
300	-35.0	-50.0	-52.27	230.481	230.482	230.596	230.768	233.06
300	-35.0	-40.0	-40.83	275.051	275.051	275.072	275.106	275.41
300	-30.0	-45.0	-47.35	231.503	231.503	231.587	231.740	234.00
300	-30.0	-35.0	-35.86	275.440	275.440	275.450	275.475	275.79
300	-25.0	-40.0	-42.43	232.486	232.486	232.541	232.676	234.91
300	-25.0	-30.0	-30.89	275.813	275.813	275.813	275.830	276.14
200	-65.0	-80.0	-81.79	148.988	148.988	149.135	149.358	151.00
200	-65.0	-70.0	-70.67	181.555	181.555	181.613	181.685	181.86
200	-60.0	-75.0	-76.87	149.837	149.837	149.985	150.187	151.81
200	-60.0	-65.0	-65.70	181.890	181.890	181.942	182.004	182.18
200	-55.0	-70.0	-71.96	150.657	150.657	150.799	150.980	152.58
200	-55.0	-60.0	-60.73	182.210	182.210	182.257	182.309	182.50
200	-50.0	-65.0	-67.04	151.449	151.449	151.579	151.741	153.33
200	-50.0	-55.0	-55.75	182.519	182.519	182.556	182.601	182.79
200	-45.0	-60.0	-62.12	152.212	152.212	152.326	152.471	154.04
200	-45.0	-50.0	-50.78	182.813	182.813	182.844	182.880	183.08
200	-40.0	-55.0	-57.19	152.947	152.947	153.043	153.172	154.72
200	-40.0	-45.0	-45.81	183.096	183.096	183.118	183.148	183.35
Max deviation from PFIT1 :					.045	.347	.811	12.75
Relative time of execution :				2	1	1	1	18

Temperature ranges for comparison (in deg C in steps of 5.0 deg)
at 1000 hPa -20 to 30
at 850 hPa -20 to 20
at 700 hPa -30 to 10
at 500 hPa -40 to 0
at 300 hPa -60 to -20
at 200 hPa -70 to -30

TABLE 7. EQUIVALENT POTENTIAL TEMPERATURE CALCULATED FROM WET-BULB POTENTIAL TEMPERATURE

WPT	EPT	EPTC	DV
40.0	478.4	478.316	-.084
38.0	454.8	454.903	.103
36.0	434.2	434.369	.169
34.0	416.5	416.315	-.185
32.0	400.5	400.394	-.106
30.0	386.2	386.306	.106
28.0	373.8	373.791	-.009
26.0	362.6	362.628	.028
24.0	352.8	352.627	-.173
22.0	343.5	343.627	.127
20.0	335.5	335.492	-.008
18.0	328.0	328.107	.107
16.0	321.3	321.375	.075
14.0	315.3	315.215	-.085
12.0	309.6	309.556	-.044
10.0	304.4	304.340	-.060
8.0	299.6	299.517	-.083
6.0	295.0	295.043	.043
4.0	290.8	290.878	.078
2.0	286.9	286.987	.087
.0	283.4	283.338	-.062
-2.0	279.9	279.902	.002
-4.0	276.7	276.651	-.049
-6.0	273.6	273.560	-.040
-8.0	270.6	270.606	.006
-10.0	267.7	267.771	.071
-12.0	265.0	265.037	.037
-14.0	262.4	262.396	-.004
-16.0	259.9	259.842	-.058
-18.0	257.4	257.381	-.019
-20.0	255.0	255.028	.028

WPT = Wet-Bulb Potential Temperature (in deg C)
EPT = Equivalent Potential Temperature (in K)
(Smithsonian Meteorological Tables)
EPTC = Equivalent Potential Temperature (Calculated);
see Appendix G
DV = EPTC - EPT

TABLE 8. WET-BULB POTENTIAL TEMPERATURE CALCULATED FROM EQUIVALENT POTENTIAL TEMPERATURE

EPT	WPT	WPTC	DV
478.4	40.0	40.000	.000
454.8	38.0	38.000	.000
434.2	36.0	35.997	-.003
416.5	34.0	34.008	.008
400.5	32.0	31.998	-.002
386.2	30.0	29.982	-.018
373.8	28.0	28.009	.009
362.6	26.0	26.003	.003
352.8	24.0	24.040	.040
343.5	22.0	21.969	-.031
335.5	20.0	19.999	-.001
328.0	18.0	17.968	-.032
321.3	16.0	15.978	-.022
315.3	14.0	14.033	.033
309.6	12.0	12.021	.021
304.4	10.0	10.027	.027
299.6	8.0	8.035	.035
295.0	6.0	5.975	-.025
290.8	4.0	3.953	-.047
286.9	2.0	1.944	-.056
283.4	0.0	0.027	.027
279.9	-2.0	-2.004	-.004
276.7	-4.0	-3.965	.035
273.6	-6.0	-5.963	.037
270.6	-8.0	-7.990	.010
267.7	-10.0	-10.038	-.038
265.0	-12.0	-12.022	-.022
262.4	-14.0	-14.004	-.004
259.9	-16.0	-15.973	.027
257.4	-18.0	-18.002	-.002
255.0	-20.0	-20.004	-.004

EPT = Equivalent Potential Temperature (in K)
WPT = Wet-Bulb Potential Temperature (in deg C) (Smithsonian Meteorological Tables)
WPTC = Wet-Bulb Potential Temperature (Calculated); see Appendix G
DV = WPTC - WPT

TABLE 9. EQUIVALENT POTENTIAL TEMPERATURE AND TEMPERATURE ON THE SATURATED ADIABAT

PRES	TEMP	DEWPT	EPT2	EPT1
1000	.0	-10.0	5.14	5.03
1000	5.0	-5.0	12.58	12.39
1000	5.0	5.0	20.13	19.80
1000	10.0	-5.0	17.75	17.60
1000	10.0	5.0	25.48	25.59
1000	15.0	-5.0	22.93	22.79
1000	15.0	5.0	30.84	30.52
1000	20.0	10.0	42.53	42.18
1000	20.0	15.0	51.09	51.27
1000	25.0	15.0	56.79	56.33
1000	25.0	20.0	68.63	68.78
1000	30.0	20.0	74.61	75.12
1000	30.0	25.0	91.05	91.04
1000	30.0	30.0	113.57	112.11
850	.0	-15.0	17.30	17.26
850	5.0	-5.0	27.56	27.39
850	10.0	.0	37.07	36.88
850	15.0	-5.0	38.46	38.30
850	15.0	5.0	48.30	48.03
850	20.0	5.0	53.97	53.66
850	20.0	15.0	72.69	72.94
700	-10.0	-30.0	19.73	19.73
700	-10.0	-20.0	21.77	21.76
700	-5.0	-25.0	26.15	26.19
700	-5.0	-15.0	29.17	29.13
700	.0	-15.0	34.83	34.83
700	.0	-5.0	41.04	40.90
700	5.0	-10.0	43.14	43.10
700	5.0	.0	51.98	52.29
700	10.0	-5.0	52.66	52.52
700	10.0	5.0	65.15	65.52
700	10.0	10.0	75.08	74.64
500	-30.0	-50.0	23.49	23.61
500	-30.0	-40.0	24.03	24.14
500	-25.0	-45.0	29.80	29.90
500	-25.0	-35.0	30.67	30.76
500	-20.0	-40.0	36.26	36.37
500	-20.0	-30.0	37.62	37.72
500	-15.0	-35.0	42.93	43.02
500	-15.0	-25.0	45.02	45.11
500	-10.0	-25.0	51.19	51.32
500	-10.0	-15.0	55.76	55.74

TABLE 9. (cont'd)

PRES	TEMP	DEWPT	EPT2	EPT1
300	-45.0	-65.0	48.64	48.86
300	-45.0	-55.0	48.85	49.08
300	-40.0	-60.0	55.77	56.01
300	-40.0	-50.0	56.15	56.37
300	-35.0	-55.0	62.97	63.22
300	-35.0	-45.0	63.60	63.84
300	-30.0	-50.0	70.28	70.48
300	-30.0	-40.0	71.31	71.55
300	-25.0	-45.0	77.75	77.95
300	-25.0	-35.0	79.42	79.66
200	-65.0	-85.0	56.36	56.70
200	-65.0	-75.0	56.39	56.70
200	-60.0	-80.0	64.29	64.62
200	-60.0	-70.0	64.34	64.66
200	-55.0	-75.0	72.22	72.57
200	-55.0	-65.0	72.33	72.67
200	-50.0	-70.0	80.18	80.53
200	-50.0	-60.0	80.38	80.73
200	-45.0	-65.0	88.17	88.51
200	-45.0	-55.0	88.53	88.86

Relative time of execution 1.4 1

Max deviation in Equivalent Potential Temperature = 1.456

Temperature TEMP ranges for comparison (in steps of 5.0 deg C):
-20 C to 30 C for 1000 hPa
-20 C to 20 C for 850 hPa
-30 C to 10 C for 700 hPa
-40 C to 0 C for 500 hPa
-60 C to -20 C for 300 hPa
-70 C to -30 C for 200 hPa
Dew-point ranges: TEMP - 20 C to TEMP in steps of 5.0 deg C

EPT1 - equivalent potential temperature by simplified formula; PTEQ1 in Appendix G
EPT2 - equivalent potential temperature by Bolton's formula; PTEQ2 in Appendix G

TABLE 10. WET-BULB POTENTIAL TEMPERATURE

PRES	TEMP	DEWPT	EPT	WPT1	WPT2	DV	WPT3	DV
1000	.0	-10.0	5.14	-2.98	-3.17	(-.19)	-3.24	(-.26)
1000	.0	-5.0	7.41	-1.61	-1.81	(-.19)	-1.82	(-.20)
1000	5.0	-5.0	12.58	1.32	1.14	(-.18)	1.12	(-.19)
1000	10.0	5.0	25.48	7.62	7.46	(-.16)	7.55	(-.07)
1000	15.0	10.0	37.03	12.23	12.11	(-.13)	12.25	(.02)
1000	20.0	10.0	42.53	14.16	14.05	(-.11)	14.32	(.16)
1000	25.0	10.0	48.03	15.94	15.84	(-.10)	16.25	(.31)
1000	25.0	15.0	56.79	18.51	18.42	(-.09)	18.83	(.32)
1000	25.0	20.0	68.63	21.56	21.48	(-.08)	21.76	(.20)
1000	30.0	15.0	62.49	20.03	19.95	(-.08)	20.57	(.54)
1000	30.0	20.0	74.61	22.94	22.87	(-.08)	23.42	(.48)
1000	30.0	25.0	91.05	26.30	26.23	(-.08)	26.58	(.28)
850	-5.0	-10.0	13.93	2.04	1.86	(-.18)	1.83	(-.21)
850	.0	-5.0	22.11	6.10	5.94	(-.16)	6.00	(-.10)
850	5.0	-5.0	27.56	8.51	8.36	(-.15)	8.46	(-.05)
850	10.0	.0	37.07	12.25	12.12	(-.13)	12.39	(.15)
850	15.0	.0	42.61	14.19	14.08	(-.11)	14.46	(.27)
850	15.0	5.0	48.30	16.02	15.92	(-.10)	16.43	(.41)
850	15.0	10.0	56.04	18.30	18.22	(-.08)	18.80	(.50)
850	20.0	10.0	61.90	19.88	19.80	(-.08)	20.62	(.74)
850	20.0	15.0	72.69	22.51	22.43	(-.08)	23.28	(.77)
700	-5.0	-15.0	29.17	9.18	9.04	(-.14)	9.06	(-.13)
700	.0	-15.0	34.83	11.42	11.29	(-.13)	11.41	(-.01)
700	.0	-5.0	41.04	13.65	13.54	(-.12)	13.89	(.24)
700	5.0	-10.0	43.14	14.37	14.25	(-.11)	14.67	(.30)
700	5.0	-5.0	46.85	15.57	15.47	(-.10)	16.02	(.45)
700	5.0	.0	51.98	17.14	17.05	(-.09)	17.76	(.62)
700	10.0	-5.0	52.66	17.34	17.24	(-.09)	18.13	(.79)
700	10.0	.0	57.91	18.82	18.74	(-.09)	19.78	(.96)
700	10.0	5.0	65.15	20.71	20.63	(-.08)	21.84	(1.13)
500	-25.0	-40.0	30.14	9.58	9.44	(-.14)	9.75	(.17)
500	-25.0	-30.0	31.47	10.11	9.97	(-.14)	10.11	(.00)
500	-20.0	-35.0	36.80	12.15	12.02	(-.13)	12.47	(.32)
500	-20.0	-30.0	37.62	12.45	12.32	(-.12)	12.69	(.25)
500	-20.0	-25.0	38.84	12.88	12.76	(-.13)	13.10	(.21)
500	-15.0	-30.0	43.77	14.57	14.46	(-.11)	15.25	(.68)
500	-15.0	-25.0	45.02	14.98	14.88	(-.11)	15.62	(.63)
500	-15.0	-20.0	46.84	15.57	15.46	(-.10)	16.22	(.65)
500	-10.0	-25.0	51.19	16.90	16.81	(-.10)	18.20	(1.30)
500	-10.0	-15.0	55.76	18.23	18.14	(-.09)	19.61	(1.38)
500	-5.0	-20.0	59.29	19.19	19.11	(-.08)	21.47	(2.28)
500	-5.0	-10.0	65.98	20.92	20.84	(-.08)	23.39	(2.47)

TABLE 10. (cont'd)

PRES	TEMP	DEWPT	EPT	WPT1	WPT2	DV	WPT3	DV
300	-50.0	-70.0	41.54	13.83	13.71	(-.11)	14.96	(1.14)
300	-45.0	-65.0	48.64	16.13	16.03	(-.10)	18.05	(1.92)
300	-40.0	-60.0	55.77	18.23	18.14	(-.09)	21.36	(3.14)
300	-40.0	-50.0	56.15	18.33	18.25	(-.09)	21.36	(3.03)
300	-35.0	-55.0	62.97	20.16	20.08	(-.08)	25.05	(4.89)
300	-35.0	-50.0	63.21	20.22	20.14	(-.08)	25.05	(4.83)
300	-35.0	-40.0	64.22	20.48	20.40	(-.08)	25.12	(4.64)
300	-30.0	-50.0	70.28	21.95	21.87	(-.08)	29.23	(7.28)
300	-30.0	-45.0	70.68	22.05	21.97	(-.08)	29.24	(7.20)
300	-30.0	-40.0	71.31	22.19	22.12	(-.08)	29.29	(7.09)
300	-30.0	-35.0	72.30	22.42	22.35	(-.08)	29.42	(7.00)
300	-25.0	-40.0	78.40	23.78	23.70	(-.08)	34.04	(10.27)
300	-25.0	-35.0	79.42	23.99	23.91	(-.08)	34.16	(10.16)
300	-25.0	-30.0	80.96	24.32	24.24	(-.08)	34.40	(10.08)
200	-60.0	-75.0	64.31	20.50	20.42	(-.08)	26.01	(5.51)
200	-60.0	-70.0	64.34	20.51	20.43	(-.08)	26.00	(5.49)
200	-60.0	-65.0	64.41	20.52	20.44	(-.08)	25.99	(5.47)
200	-55.0	-75.0	72.22	22.40	22.33	(-.08)	30.89	(8.49)
200	-55.0	-70.0	72.26	22.41	22.34	(-.08)	30.89	(8.47)
200	-55.0	-60.0	72.46	22.46	22.38	(-.08)	30.87	(8.41)
200	-50.0	-70.0	80.18	24.15	24.07	(-.08)	36.54	(12.39)
200	-50.0	-60.0	80.38	24.19	24.12	(-.08)	36.52	(12.33)
200	-50.0	-55.0	80.60	24.24	24.16	(-.08)	36.51	(12.27)
200	-45.0	-65.0	88.17	25.76	25.68	(-.08)	43.02	(17.26)
200	-45.0	-55.0	88.53	25.83	25.75	(-.08)	43.00	(17.17)
200	-45.0	-50.0	88.91	25.90	25.82	(-.08)	42.99	(17.09)

	WPT2 DV	WPT3 DV
Max deviation at 1000 hPa =	.26	.54
Max deviation at 850 hPa =	.23	.77
Max deviation at 700 hPa =	.22	1.24
Max deviation at 500 hPa =	.18	6.10
Max deviation at 300 hPa =	.15	10.37
Max deviation at 200 hPa =	.10	17.26

Relative time of execution (overall procedures)

WPT1	WPT2	WPT3
1	16.3	1.2

DV - Deviation

Deviation compared with temperature TEMP ranges (in steps of 5 deg):

-20 C to 30 C for 1000 hPa
-20 C to 20 C for 850 hPa
-30 C to 10 C for 700 hPa
-40 C to 0 C for 500 hPa
-60 C to -25 C for 300 hPa
-70 C to -45 C for 200 hPa

and dew-point ranges: TEMP - 20 C to TEMP in steps of 5 deg C

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REFERENCES

1. Betts, A.K. and Dugan, F.J.	1973	"Empirical Formula for Saturation Pseudoadiabats and Saturation Equivalent Potential Temperature", J. Appl. Meteor., 12 p.731.
2. Bolton, D.	1980	"The Computation of Equivalent Potential Temperature", Mon. Wea. Rev., 108 p.1046.
3. Holton, J.R.	1979	An Introduction to Dynamic Meteorology, Academic Press.
4. Hull, A.N.	1974	"Comments on 'A Simple but Accurate Formula for the Saturation Vapor Pressure Over Liquid Water'", J. Appl. Meteor., 13 p.607.
5. List, R.	1963	Smithsonian Meteorological Tables 6th edition (1958), 2nd reprint. Smithsonian Institute.
6. McIntosh, D.H. and Thom, A.S.	1978	Essentials of Meteorology, Wykeham Publications.
7. Murray, F.W.	1967	"On the Computation of Saturation Vapor Pressure", J. Appl. Meteor., 6 p.203.
8. Profs Program Office.	1982	Profs Library. Available from NOAA Enviromnent Research Lab, Boulder, Colorado
9. Riegel, C.A.	1974	"Comments on 'A Simple but Accurate Formula for the Saturation Vapor Pressure Over Liquid Water'", J. Appl. Meteor., 13 p.606.
10. Sargent, G.P.	1980	"Computation of vapour pressure, dew-point and relative humidity from dry and wet-bulb temperatures", Met. Mag., 109 p.238.
11. Saunders, P.M.	1975	"The Thermodynamics of Saturated Air: A Contribution to the Classical Theory", Quat. J. Roy. Meteor. Soc., 83 p.342.
12. Shen, C. 沈春康	1983	大氣熱力學 Atmospheric Thermodynamics, 氣象出版社 Meteorological Publications.

13.	Simpson, R.H.	1978	"On the Computation of Equivalent Potential Temperature", Mon. Wea. Rev., <u>106</u> p.124.
14.	Tabata, S.	1973	"A Simple but Accurate Formula for the Saturation Vapor Pressure Over Liquid Water", J. Appl. Meteor., <u>12</u> p.1410.
15.	U.K. Meteor. Office (ed.)	1964	Hygrometric Tables, Part II, 2nd edition. HMSO.
16.	Wallace, J.M. and Hobb, P.V.	1977	Atmospheric Science, An Introductory Survey, Academic Press.
17.	Wexler, A.	1976	"Vapor Pressure Formulation for Water in Range 0 to 100 °C. A Revision", J. Res. Nat. Bur. Stand., <u>80A</u> p.775.
18.	Wigley, T.M.L.	1974	"Comments on 'A Simple but Accurate Formula for the Saturation Vapor Pressure Over Liquid Water'", J. Appl. Meteor., <u>13</u> p.608.
19.	Wilhelmson, R.B.	1977	"On the Thermodynamic Equation for Deep Convection", Mon. Wea. Rev., <u>105</u> p.545.
20.	WMO	1966	WMO International Meteorological Tables, No.188 TP.94.
21.	WMO	1979	WMO Technical Regulation, No.49.
22.	Wurtele, M.G. and Finke, B.	1961	"A Note on the Computation of the Saturation- and Pseudo-Adiabatic Processes", J. Meteor.,<u>18</u>.
23.	Yang, D. 楊大升 Liu, Y. 劉余濱 and Liu, S. 劉式適	1983	動力氣象學 氣象出版社 Dynamic Meteorology, Meteorological Publications.
24.	Zhongyang Qixianju (ed.) 中央氣象局(編)	1980	濕度查算表 氣象出版社 Psychrometric Tables, Meteorological Publications

APPENDIX A
PHYSICAL CONSTANTS

1. Molecular weight of water vapour 18.0153 $\mathrm{g\ mol^{-1}}$
2. Apparant molecular weight of dry air 28.9644 $\mathrm{g\ mol^{-1}}$
3. Molar gas constant for ideal gas R 8.31432 $\mathrm{J\ mol^{-1}\ K^{-1}}$
4. Gas constant for dry air R_d 287.05 $\mathrm{J\ kg^{-1}\ K^{-1}}$
5. Gas constant for water vapour R_v 461.51 $\mathrm{J\ kg^{-1}\ K^{-1}}$
6. Specific heat capacity for dry air at constant pressure C_{pd}

   Range of values: 1003 to 1011 $\mathrm{J\ kg^{-1}\ K^{-1}}$

   Values used in computation: 1005 $\mathrm{J\ kg^{-1}\ K^{-1}}$
7. Specific heat capacity for liquid water C_w

   Range of values: 4180 to 4770 $\mathrm{J\ kg^{-1}\ K^{-1}}$

   Values used in computation: 4190 $\mathrm{J\ kg^{-1}\ K^{-1}}$
8. Latent heat of vaporization L_v

   Range of values: 2.635 to 2.406×10^6 $\mathrm{J\ kg^{-1}}$

   Values used in computation: 2.501×10^6 $\mathrm{J\ kg^{-1}}$

APPENDIX B

LIST OF SYMBOLS

Symbol	Unit	Description
c_p	J kg^{-1} K^{-1}	specific heat capacity of air parcel at constant pressure
c_{pd}	J kg^{-1} K^{-1}	specific heat capacity for dry air at constant pressure
c_{pm}	J kg^{-1} K^{-1}	specific heat capacity for moist air at constant pressure
c_{pv}	J kg^{-1} K^{-1}	specific heat capacity for water vapour at constant pressure
e	hPa	vapour pressure of air parcel
e_s	hPa	saturation vapour pressure
L_v, γ	J kg^{-1}	latent heat of vaporization
p	hPa	pressure
P_{LCL}	hPa	pressure at the lifting condensation level
r, r_w	g kg^{-1}	mixing ratio
R_d	J kg^{-1} K^{-1}	gas constant for dry air
R_m	J kg^{-1} K^{-1}	gas constant for moist air
R.H.,RH	%	relative humidity
S.H.	-	specific humidity
T, TT	°C	dry-bulb temperature
T_d	°C	dew-point temperature
T_{LCL}	°C	temperature at the lifting condensation level
T_w	°C	wet-bulb temperature
ε		ratio of molar mass of dry air to that of water
Γ	°C m^{-1}	lapse rate for dry air
Γ_m	°C m^{-1}	lapse rate for moist air
θ	°C	potential temperature
θ_d	°C	potential temperature for dry air portion
θ_e	°C	equivalent potential temperature
θ_w	°C	wet-bulb potential temperature

All temperature parameters are measured in degrees Celcius unless specified otherwise, when these parameters are measured in Kelvin.

APPENDIX C

LIST OF FORMULAE FOR SATURATION VAPOUR PRESSURE OVER WATER

1. Goff-Gratch formula (List 1963, Murray 1967) GG1

T_s = 373.16 K (Steam Point)

Temperature in Kelvin

$$A = \frac{T_s}{T} - 1$$

$$B = 1 - \frac{T}{T_s}$$

$$\begin{aligned}\log_{10} e_s = {} & -7.90298\ A + 5.02808 * \log_{10}(A + 1) \\ & - 1.3816 \times 10^{-7} * (10^{11.344\ B} - 1) \\ & + 8.1328 \times 10^{-3} * (10^{-3.49149\ A} - 1) \\ & + \log_{10}(1013.246)\end{aligned}$$

2. Goff-Gratch formula (中央氣象局 1980, WMO 1966 & 1979) GG2

T_r = 273.16 K

Temperature in Kelvin

$$A = 1 - \frac{T_r}{T}$$

$$B = \frac{T}{T_r} - 1$$

$$\begin{aligned}\log_{10} e_s = {} & 10.79574 * A - 5.028 * \log_{10}(B + 1) \\ & + 1.50475\text{E-}4 * (1 - 10^{-8.2969\ B}) \\ & + 0.42873\text{E-}3 * (10^{4.76955\ A} - 1) + 0.78614\end{aligned}$$

3. Wexler's formula (Wexler 1976, Bolton 1980) WEX

Temperature T measured in Kelvin

$$\begin{aligned}\ln(100\ e_s) = {} & 18.87643854 - 2.8354721 \times 10^{-2}\ T \\ & + 1.7838301 \times 10^{-5}\ T^2 - 8.4150417 \times 10^{-10}\ T^3 \\ & + 4.4412543 \times 10^{-13}\ T^4 - 6.0170128 \times 10^{3}\ \frac{1}{T} \\ & - 2.9912729 \times 10^{3}\ \frac{1}{T^2} + 2.858487 * \ln(T)\end{aligned}$$

4. Sargent's formula (Sargent 1980) SG

$$\begin{aligned}\ln(e_s) = {} & 1.809567918 + 0.07266296315\ T \\ & - 0.2996403370\text{x}10^{-3}\ T^2 + 0.1160464233\text{E-}5\ T^3 \\ & - 0.4606513971\text{x}10^{-8}\ T^4 + 0.2315159066\text{E-}10\ T^5 \\ & - 0.1103513356\text{x}10^{-12}\ T^6\end{aligned}$$

5. Simplified Wexler's formula (Bolton 1980) SW1

$$e_s = 6.112 * \exp\left(17.67 * \frac{T}{T + 243.5}\right)$$

6. Simplified Wexler's formula (Bolton 1980) SW2

$$\begin{aligned}e_s = {} & 1.7743 \times 10^{-8}\ (T + 273.15)^{3.504} \\ & \exp\left(12.992\ \frac{T}{T + 217.8}\right)\end{aligned}$$

7. Teten's formula (Bolton 1980) TT1

$$e_s = 6.1078 * 10^{\left(7.5 * \frac{T}{T + 237.3}\right)}$$

8. Teten's formula (Riegel 1974) TT2

$$e_s = 6.1078 * 10^{\left(7.567 * \frac{T}{T + 239.7}\right)}$$

9. Wobus' formula (Profs Library 1982) WOB

$$\begin{aligned}V = {} & 0.99999683 - 0.90826951 \times 10^{-2}\ T \\ & + 0.78736169 \times 10^{-4}\ T^2 - 0.961117958 \times 10^{-6}\ T^3 \\ & + 0.43884187 \times 10^{-8}\ T^4 - 0.29883885 \times 10^{-10}\ T^5 \\ & + 0.21874425 \times 10^{-12}\ T^6 - 0.17892321 \times 10^{-14}\ T^7 \\ & + 0.11112018 \times 10^{-16}\ T^8 - 0.30994571 \times 10^{-19}\ T^9\end{aligned}$$

$$e_s = 6.1078\ \frac{1}{V^8}$$

10. Norquist's formula (Norquist 1973 re: Profs Library) NQ

Temperature in Kelvin

$$A = 11.344 - 0.0303998\ T$$

$$B = 3.49149 - 1302.8844\ \frac{1}{T}$$

$$C = 23.832241 - 5.02808\ \log_{10}\ T$$

$$e_s = 10^{\left\{ C - 1.3816 \times 10^{-7} * 10^{A} + 8.1328 \times 10^{-3}\ 10^{B} - 2949.076\ /\ T \right\}}$$

11. Richards' formula (Wigley 1974) RC

Temperature in Kelvin

$$A = 1 - \frac{373.15}{T}$$

$$B = 13.3185\ A - 1.976\ A^2 - 0.6445\ A^3 - 0.1299\ A^4$$

$$e_s = 1013.25 * \exp\ (B)$$

12. Lowe's formula (Lowe 1977) LW

$$\begin{aligned} e_s = {} & 6.107799961 + 4.436518521 \times 10^{-1}\ T \\ & + 1.428945805 \times 10^{-2}\ T^2 + 2.650648471 \times 10^{-4}\ T^3 \\ & + 3.031240396 \times 10^{-6}\ T^4 + 2.034080948 \times 10^{-8}\ T^5 \\ & + 6.136820929 \times 10^{-11}\ T^6 \end{aligned}$$

13. Berry et.al's formula (Berry 1945 re: Tabata 1973) 3B

$$A = \frac{273}{T + 273.15}$$

$$e_s = 6.105\ A^{5.31}\ \exp\ (25.22\ (1-A))$$

14. Tabata's formula (Tabata 1973) TB1

$$e_s = 10^{\left(9.28603523 - \frac{2.32237885 \times 10^3}{TT+273.15} \right)}$$

15. Tabata's formula (Tabata 1973) TB2

$$A \quad \frac{1000}{T + 273.15}$$

$$e_s = 10^{8.42926609 - 1.82717843\ A - 0.071208271\ A^2}$$

APPENDIX D

LIST OF FORMULAE FOR SATURATION VAPOUR PRESSURE OVER ICE

1. Goff-Gratch formula (List 1963, Murray 1967) GG1

Temperature in Kelvin

$$A = \frac{273.15}{T}$$

$$B = 1 - \frac{T}{273.15}$$

$$\log_{10} e_s = - 9.09718\ (A - 1) - 3.56654 \log_{10} (A) + 0.876793\ B + \log_{10} (6.1071)$$

2. Goff-Gratch formula (中央氣象局 1980, WMO 1966 & 1979) GG2

$T_r = 273.16$ K

Temperature in Kelvin

$$A = \frac{T_r}{T}$$

$$\log_{10} e_s = -9.09685 * A - 3.56654 * \log_{10} (A) - \frac{0.87682}{A} + 10.75981$$

3. Lowe's formula (Lowe 1977) LW

$$\begin{aligned} e_s = {} & 6.109177956 + 5.034698970 \times 10^{-1}\ T \\ & + 1.886013408 \times 10^{-2}\ T^2 + 4.176223716 \times 10^{-4}\ T^3 \\ & + 5.824720280 \times 10^{-6}\ T^4 + 4.838803174 \times 10^{-8}\ T^5 \\ & + 1.838826904 \times 10^{-10}\ T^6 \end{aligned}$$

APPENDIX E

LIST OF FORMULAE FOR CALCULATING TEMPERATURE FROM SATURATION VAPOUR PRESSURE OVER WATER

1. Sargent's formula (Sargent 1980) SG

$$V = \ln(e_s)$$

$$T = -22.59529963 + 11.33418988\,V + 0.5756940348\,V^2 + 0.3025080051 \times 10^{-1}\,V^3 + 0.1778276954 \times 10^{-2}\,V^4 + 0.7443287646 \times 10^{-4}\,V^5 + 0.1129170314 \times 10^{-4}\,V^6$$

2. Bolton's formula (Bolton 1980) BT

$$V = \ln(e_s)$$

$$T = \frac{243.5\,V - 440.8}{19.48 - V}$$

3. Inverse Teten's formula (Profs Library 1982) ITT

$$X = \log_{10}\left(\frac{e_s}{6.1078}\right)$$

$$B = 7.5 - X$$

$$T = 237.3\,\frac{X}{B}$$

APPENDIX F

LIST OF FORMULAE FOR CALCULATING TEMPERATURE AND PRESSURE AT THE LIFTING CONDENSATION LEVEL

1. TLCL1 (Bolton 1980)

Temperature in Kelvin

$$T_{LCL} = \frac{2840}{3.5 \ln(T) - \ln(e_s(T_d)) - 4.805} + 55$$

2. TLCL2 (Bolton 1980)

Temperature in Kelvin

$$A = \frac{1}{T_d - 56}$$

$$B = \frac{1}{800} \ln\left(\frac{T}{T_d}\right)$$

$$T_{LCL} = \frac{1}{A+B} + 56$$

3. TLCL3 (Wobus re: Profs Library 1982)

$S = T - T_d$ = dew-point depression

$$T_{LCL} = T - 1.2185\ S + 1.278 \times 10^{-3}\ T \cdot S - 2.19 \times 10^{-3}\ S^2$$
$$+ 1.173 \times 10^{-5}\ S^3 - 5.2 \times 10^{-6}\ T\ S^2$$

4. TLCL4 (List 1963)

$$A = \frac{\log_{10}(T)}{0.286} - \log_{10}(e_s(T_d))$$

$$T_{LCL} = 5.2811771161 \times 10^3 - 2.7855244942 \times 10^3\ A$$
$$+ 6.1825383673 \times 10^2\ A^2 - 7.1113206422 \times 10^1\ A^3$$
$$+ 4.1693969217\ A^4 - -9.8966217588 \times 10^{-2}\ A^5$$

5. TLCL5 (List 1963, Bolton 1980)

$$A = \frac{e_s(T_d)}{T^{3.5011}}$$

$$B = \ln(A) + 17.84727477$$

$$T_{LCL} = \frac{217.8\ B}{12.992 - B}$$

6. Poisson's equation with moisture included PLCL1

$$P_{LCL} = p \left(\frac{T_{LCL} + 273.15}{T + 273.15} \right)^{\frac{3.5011}{1 - 0.000293r}}$$

7. Poisson's equation without including moisture PLCL2

$$P_{LCL} = p \left(\frac{T_{LCL} + 273.15}{T + 273.15} \right)^{3.5011}$$

8. PTLCL (Schlatter 1981 re: Profs Library)

Approximate lines of constant potential temperature and constant mixing ratio on the skew T - log P chart by straight lines

$$A = \frac{4098.026}{(T_d + 237.3)^2}$$

$$B = \frac{1}{0.28541\ (T + 273.15)}$$

$$P_{LCL} = P * \exp \frac{A\ B\ (T - T_d)}{B - A}$$

$$T_{LCL} = T + A\ \frac{T - T_d}{B - A}$$

APPENDIX G

LIST OF ALGORITHMS FOR CALCULATING EQUIVALENT AND WET-BULB POTENTIAL TEMPERATURES, AND TEMPERATURE AND PRESSURE ON THE SATURATION ADIABAT

1 .PTEQ1 (Yang et al 1980, Wallace and Hobb 1977)

Temperature in Kelvin;
Air parcel saturated with water vapour

$$\theta_e = T \left(\frac{1000}{pp} \right)^{0.286} \exp \left(\frac{B\, r_w}{T} \right)$$

$B = 2.6518986 \cong L_v$

(If air parcel is not saturated, lift the air parcel up to the lifting condensation level and use T_{LCL} instead of T.)

2. PTEQ2 (Bolton 1980)

Temperature in Kelvin

$$\theta_e = T \left(\frac{1000}{p} \right)^{0.2854(1-0.297\times10^{-3}r)}$$

$$\times \exp \left[\left(\frac{3.376}{T_{LCL}} - 0.00254 \right) r \left(1 + 0.81\times10^{-3}r\right) \right]$$

3. PTEQ3 Calculate equivalent potential temperature from wet-bulb potential temperature
(Formula obtained by fitting the parameters in table 78, List 1963)

$$\theta_e = 10.18804873 + 1.7688694162\, \theta_w + 2.6549216624 \times 10^{-2}\, \theta_w^{\,2} + 5.9275582411 \times 10^{-4}\, \theta_w^{\,3} + 5.6218257129 \times 10^{-6}\, \theta_w^{\,4} + 3.5189108876 \times 10^{-8}\, \theta_w^{\,5} + 6.3019127555 \times 10^{-9}\, \theta_w^{\,6}$$

4. WPT1 Calculate wet-bulb potential temperature from equivalent potential temperature
(Formula obtained by fitting the parameters in table 78, List 1963)

$$\begin{aligned}\theta_w = & - 6.2609512839 + 6.6480400261 \times 10^{-1}\,\theta_e \\ & - 5.1338815795 \times 10^{-3}\,\theta_e^{\,2} + 8.1910107184 \times 10^{-6}\,\theta_e^{\,3} \\ & + 4.5363160786 \times 10^{-7}\,\theta_e^{\,4} - 6.3992885228 \times 10^{-9}\,\theta_e^{\,5} \\ & + 4.0670460222 \times 10^{-11}\,\theta_e^{\,6} - 1.2831483168 \times 10^{-13}\,\theta_e^{\,7} \\ & + 1.6177730539 \times 10^{-16}\,\theta_e^{\,8}\end{aligned}$$

5. WPT2 Wet-bulb potential temperature
(Stipanuk 1973 re: Profs Library) WPT2
step i. Calculate equivalent potential temperature with PTEQ2
ii. Calculate wet-bulb potential temperature with TTSA1

6. WPT3 Wet-bulb potential temperature (Profs Library)

$$\theta_w = \theta + 6.1071 \left[\exp\left(\frac{T}{f(T)}\right) - \exp\left(\frac{T_d}{f(T_d)}\right) \right]$$

where

$$\begin{aligned}f(y) = & \ 1.8199427 \times 10^{1} + 2.16408 \times 10^{-1}\,y \\ & + 3.071631 \times 10^{-4}\,y^2 + 3.895366 \times 10^{-6}\,y^3 \\ & + 1.96182 \times 10^{-8}\,y^4 + 5.293557 \times 10^{-11}\,y^5 \\ & + 7.399595 \times 10^{-14}\,y^6 - 4.19835 \times 10^{-17}\,y^7\end{aligned}$$

7. TTSA1 Temperature on a pseudoadiabat defined by an equivalent potential temperature (Yang et al. 1983, Wallace and Hobb 1977)

step i. initial guess for T

ii. calculate θ_e using PTEQ1

iii. adjust T until deviation from given θ_e is small

8. TTSA2 Temperature on a pseudoadiabat defined by an equivalent potential temperature

Similar to TTSA1 but use PTEQ2

9. PPSA1 Pressure on a pseudoadiabat defined by an equivalent potential temperature

Similar to TTSA1 but calculate pressure from given temperature instead of calculate temperature from given pressure

APPENDIX H
OTHER FORMULAE

1. Mixing ratio without correction for deviation from ideal gas

$$r = \varepsilon \frac{e}{p - e}$$

$$\varepsilon = 0.62198$$

2. Mixing ratio with correction for deviation from ideal gas

$$r = \varepsilon \frac{fe}{p - fe}$$

$$f = 1 + 4.5 \times 10^{-6} p + 5.6 \times 10^{-7} \left(T - 12.5 + \frac{7500}{p}\right)^2$$

$$\varepsilon = 0.62198$$

The function f fits table 89, List 1963 by Wobus (Profs Library)

3. Relative humidity

$$R.H. = 100\,\% \times \frac{e_s(T_d)}{e_s(T)}$$

4. Temperature from saturation mixing ratio and pressure

(Stipanuk 1973 re: Profs Library)

$$y = \log_{10} \left(r \frac{p}{622 + r}\right)$$

$$T = 10^{(0.0498646455\, y + 2.4082965)} - 7.07475$$

$$+ 38.9114 \times 10^{(0.0915\, y - 1.2035)^2} - 273.15$$

5. Dew-point from relative humidity and temperature

(Parry 1969 re: Profs Library)

$$y = 1. - 0.01 * (R.H.)$$

$$T_d = T - (14.55 + 0.114\ T)\ y - ((2.5 + 0.007\ T)\ y)^3 - (15.9 + 0.117\ T)\ y^{14}$$

6. Specific humidity

$$S.H. = \frac{r}{1 + r}$$