ROYAL OBSERVATORY, HONG KONG TECHNICAL NOTE NO. 58

A DESIGN RAIN STORM PROFILE FOR HONG KONG

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1. INTRODUCTION

Engineers use a number of different methods to calculate the optimum size of drains. The method most commonly used in Hong Kong is the so called "Rational Method" (Road Research Laboratory Note 35, 1963 and Manual of Sewerage and Drainage Practice Section 2). This method provides a means of estimating the pipe size from the peak rate of runoff and the "time of concentration" which is the time of flow in the longest pipe in the catchment area under consideration plus an empirical allowance for the time of entry. The peak rate of runoff is in turn calculated from the area of impermeable surface in the catchment and the rate of rainfall corresponding to a duration equal to the "time of concentration" for a given frequency of occurrence.

In Hong Kong appropriate rainfall information has been taken from Table 12, "Expected annual extreme rainfall intensity (mm h⁻¹) for specified time intervals and return periods", in Royal Observatory Tech. Note No. 24 (S. Cheng and W.H. Kwok, 1966) using data during 1952-1965. An expanded version appears as Table 7.3 in Royal Observatory Tech. Mem. No. 10 (Bell and Chin, 1968). The latest updated version is given as Table II in this publication using all available data up to and including 1980. Tables III and IV provide alternative methods of calculating return periods after Jenkinson (1977).

The frequency of occurrence mentioned above should correspond to the frequency at which flooding can be tolerated in the area under consideration. This depends very much on whether flooding will endanger life and to a lesser extent on economic considerations such as damage to foundations, washouts of roads etc.

The "Rational Method", widely used as it may be, is often criticized for its inaccuracy when applied to larger catchments. In Hong Kong, various firms of consulting engineers have suggested that the method leads to uneconomical overdesign of drainage systems in large catchments, particularly those in the northwestern part of the New Territories where the "time of concentration" is about 4 hours which is much longer than the effective life of many rainstorms.

An alternative method which has been suggested for use in Hong Kong is the Transport and Road Research Laboratory Hydrograph Method (Road Research Lab. Note 35, 1963). This method has been shown to provide more accurate and reliable designs in larger areas but it requires an estimate of the profiles of design storms for various return periods in order to calculate the flow. This paper is an attempt to produce usable storm profiles for Hong Kong and to answer some of the questions most frequently raised by drainage engineers concerning Hong Kong's rainfall.

2. RAINFALL DATA

A rain storm profile is a time series of the rate of rainfall in a storm, lasting a few hours. The profile under study is not that of a particular historical storm, but that of an extreme storm associated with a particular return period. The annual rainfall maximum for various time durations are extracted from three types of rainfall records for analysis. The first type is the hourly rainfall measured at the end of every clock hour using an ordinary raingauge. The longest and most complete set of data (1889-1939, 1947 onwards) is recorded at the headquarters of the Royal Observatory. The second type is the continuous rainfall recorded on autographic charts. Although there are Heath and Hicks automatic recorders in operation. the longest records are from the headquarters of the Observatory where a Casella tilting siphon raingauge has been in use since 1947. The highest resolution obtainable from these raingauges is 15 minutes. The third type is from the Jardi instantaneous rate-of-rainfall recorder and the one installed at the King's Park Meteorological Station in 1952 has the longest period of records. The response time of this instrument is approximately 15 seconds (T.T. Cheng, 1971). Table I shows the types of rainfall records used for various durations.

In order to provide background information for the selection of appropriate frequencies, a list of recent rainstorms in Hong Kong, their effects and the extent of flooding associated with them is included in Appendix I. However, the heaviest rainstorms on record occurred earlier. Chan (1976) describes how 697.1 mm fell in 24 hours and 841.3 mm in two days on 29-30 May 1889; and 505.1 mm fell in the 8 hours 2 a.m. - 10 a.m. on 19 July 1926.

Calculation of Return Periods

Gumbel's Method

In previous studies such as Royal Observatory Tech. Note No. 24 and Tech. Mem. No. 10, Gumbel's statistics of extremes (Gumbel 1954) was used to calculate the extreme depths or intensities of rainfall for specified return periods. Table II is an updated version of this previous work. In Table II in order to present the results in their most usable form, extreme depths and extreme intensities are tabulated for long and short durations respectively. Extreme depths can be easily converted to extreme intensities in appropriate units by dividing by the duration. The parameters, μ and 1/4, are characteristic of each duration, and the extreme depths or intensities can be calculated, for a given duration and return period from the following formulae:-

$$P = 1 - \frac{1}{T} \tag{1}$$

$$Y == -ln (-lnF)$$
 (2)

$$X = \mu + \frac{1}{d} Y \tag{3}$$

where T is the return period in years.

X is the extreme depth in mm or intensity in mm h^{-1} respectively.

P is the probability of an annual maximum being equal to or less than X.

Y is called the reduced variate.

It should also be noted that there are no actual rainfall records for durations of 30 seconds, 60 seconds, 2 minutes, 5 minutes, 10 minutes and 18 hours in Table II. The figures tabulated are calculated from interpolated values of μ and 1/a. The values of μ and 1/a were smoothed graphically by plotting the calculated values against rainfall duration.

Jenkinson's Method

However, a survey of the literature shows that Gumbel's method is not necessarily the best method (Gringorten (1962) and Servuk & Geiger (1981)). A more general three parameter model which includes Gumbel as a special case has been developed by Jenkinson (1977). When the annual extremes are plotted on extreme probability paper the points may lie on a straight line. In this case the parameter k is zero and the data fit a Fisher-Tippett Type I or Gumbel distribution. If the points lie on a curve which is concave upwards then k is negative and the distribution is of the Fisher Tippett Type II or Frechet distribution. If the curve is concave downwards then k is positive and the distribution is of the Fisher Tippett Type III or Weibull distribution. Jenkinson's methods are basically methods of curve fitting.

Computer programs were written to fit the observed distribution of annual extremes into Jenkinson's three parameter distribution and hence to predict the extreme values for various return periods. The results are given in Tables III and IV. In compiling Table III Jenkinson's 1955 solution was used to estimate the three parameters. Both the annual and the biennial extremes are used to estimate the three parameters in this method. Jenkinson's maximum likelihood solution (1977) was used to estimate the three parameters in Table IV. In Tables III and IV the parameters X, and k were smoothed graphically by plotting the calculated parameters against rainfall duration.

In order to choose the correct "return period" for a particular design it is important to evaluate the risks involved. Figure I shows the probabilities "Pr" that a particular level with a return period "T" will be reached or exceeded at least once in a period "r". For example, if an extreme event has a return period of 100 years there is nearly a one in ten risk (Pr = 0.0956) that the event will occur at least once in the next 10 years and a fifty-fifty chance (Pr = 0.5) that it will occur within 69 years. There are roughly two chances in three (Pr = 0.63) that it will occur within 100 years. Figure 1 is calculated from the formula (Hersfield, 1973).

$$Pr = 1 - (1 - \frac{1}{T})^{T}$$
 (4)

3. THE STORM PROFILE

In Table II, for a given return period, the maximum depths (or intensities) for various durations occur with the same frequency but they do not necessarily occur in the same storm. If all the maxima occur in one storm, that storm should represent the worst one for that return period.

When the extreme intensity is plotted against duration, a typical plot is obtained as shown in Figure 2. The shaded area represents the extreme depth of rainfall in duration 't' for that particular return period. With the assumption of symmetry, a profile for the same return period is constructed under the constraint that the maximum amount of rain given by the profile for any duration equals the extreme depth given by Figure 2. The same contraint can be expressed in another way: if Figure 3 represents a symmetric storm profile for the same return period with the peak of the profile occurring at time = 0, the shaded area in Figure 3 should be equal to the shaded area in Figure 2 for any value of t_d.

For computation purposes, it is convenient to approximate the extreme intensity vs duration curve in Figure 2 by a formula of the form (Wisner, 1981)

$$I = \frac{a}{(t+b)^c} \tag{5}$$

where

I is the extreme intensity in mm h^{-1}

t is the duration in minutes

a, b and c are constants depending on the return period

The method used in this publication to determine a, b and c is outlined in Appendix II. The formula for the storm profile can be derived from equation (5) under the above-mentioned contraint as:

$$F(t) = \frac{a[b+2(1-c)t]}{(2t+b)^{c+1}} \text{ for } t \ge 0$$
 (6)

where

F(t) is the rate of rainfall in mm h^{-1}

t is the time in minutes

a, b, c are the same constants as in equation (5).

The mathematical details of derivation can be found in Appendix III.

* An alternative form of (5) is $I = \frac{I_0}{(1+\frac{t}{b})^c}$ where $I_0 = \frac{a}{b^c}$ is the

instantaneous rainfall intensity.

4. CALCULATION OF STORM PROFILES

Based on Table II, expected extreme intensity is plotted against duration for each of the return periods and the data points are fitted with curves of the form in Eqn. 5 (Appendix II). The calculated values of a, b and c in Eqn. 5 are given in Table V. Table VI depicts the goodness of fit by giving the percentage deviation of the data from the fitted curves.

Fig. 4 shows a fitted curve corresponding to Eqn. 5 for a return period of 20 years (Gumbel).

The storm profiles constructed according to Eqn. 6 are plotted in Figs. 5-7 corresponding to the three different distributions of extreme values.

These rainstorm profiles are similar to those calculated by the U.K. Meteorological Office and quoted in Road Note No. 35 (Road Research Laboratory 1963). For purposes of comparison Fig. 8 shows storm profiles used in the U.K. Apart from the obvious differences in intensity, the U.K. profiles tend to die off more rapidly than the corresponding ones for Hong Kong. However the time taken for the intensity to diminish to half its peak value is approximately the same.

5. DISCUSSION & CONCLUSION

The analyses in this paper rely solely on the annual rainfall maxima. There are inherent shortcomings in using only annual extremes. Firstly, a lot of rainfall data is not used. Secondly, extreme values are notoriously prone to error. For example, failure of instruments in heavy rainstorms may easily result in erroneous extremes. Thirdly, in some years destructive rainstorms are ignored because they are not the heaviest in the year while in other years extreme values are included although they are relatively insignificant. Finally, assumptions have to be made about the distribution of the extremes in order to estimate the values for various return periods.

It is still debatable which distribution is best for extrapolation. Gumbel's method assumes that future extremes will lie on the best straight line through the available data and is therefore least affected by an unrepresentative sample. The 3-parameter methods assume that some of the curvature will persist and they are therefore better when the sample is larger and likely to be representative of the future. As expected Table II, III and IV look very similar. The parameter k varies with the rainfall duration and even changes sign, indicating that extreme rainfall does not always belong to any one particular type of distribution. There is generally good agreement for small return periods and it is only for long return periods that differences become significant.

One of the basic assumptions in this analysis is the symmetry of the profile. However, this is not always realistic. For example, it has been observed, that the precipitation is heaviest shortly after the start and dies away gradually towards the end of a thunderstorm (Byers & Braham, 1953, University of Chicago). An asymmetric profile has been developed by Keifer and Chu (1957) for Chicago. However, they point out that the symmetric design rainstorm leads to more conservative design than a storm in which the heaviest rainfall occurs near the beginning when more of the water is absorbed in surface depressions (puddles) or in unsaturated soil. Their asymetric profile requires the preselection of a parameter 'r' which is the ratio of the time before the peak of the storm to the total duration. This parameter has to be determined empirically using data from historical storms. However, examination of the storm profiles given on pages 88-100 of Bell & Chin (1968) shows so much diversity that no realistic value of 'r' could be chosen. Therefore, the simpler symmetric profile was selected.

Another point is that the original rainfall data used in the calculations were recorded at the headquarters of the Royal Observatory and at nearby King's Park Meteorological Station. Although the rainfall at the Observatory has been found to be fairly typical of Hong Kong, the application of the profile developed in this paper to locations other than the Observatory may not be valid. A map showing the 25-year mean rainfall distribution in Hong Kong is included in Figure 9 as an indication of the variation from place to place.

6. ACKNOWLEDGEMENT

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Note: More references on rainfall in Hong Kong are available in the Royal Observatory Publications List.

TABLE I. TYPES OF RAINFALL RECORDS USED IN THIS NOTE

Duz	rations	Types of record used
15	seconds	3
15	minutes	2
30	11	2
60	**	1 , 2 ⁺
2	hours	1 , 2 ⁺
4	11	1
6	**	1
8	••	1
12	**	1
24	11	1
2	days	1
3	11	1
4	11	1
5	**	1
7	**	1
15	**	1
31	*	1

Type 1: hourly records measured at the Royal Observatory (1884-1939; 1947-1980)

Type 2 : continuous rainfall recorded at the Observatory (1947-1980)

Type 3: instantaneous rate-of-rainfall records at King's Park (1952-1980)

Type 1 gives hourly rainfall at clock hours and daily rainfall ending at mid-night while type 2 gives rainfall for any 60 minutes, 120 minutes etc. However pre-war rainfall measurements were made at half past each hour so that hourly rainfall was centred on the clock hour.

EXTREME DEPTH OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS (GUMBEL) $X = \mu + \frac{1}{2}X$ TABLE II.

Midia	Contract				DEWIND DEPT (VEARS)	PION (VEA	PC)			
41	FARAMETERS			-	THE TANK	מבון מחוו	(2)			
	1/ھ	2	5	10	20	50	100	200	500	1000
	mm	mm	E		ww	ana a	mm	TIEST TERM	ana a	1
	157.449	899	847	965	1080	1220	1330	1440	1590	1700
	141,297	484	644	750	851	983	1080	1180	1310	1410
	120.418	360	496	587	673	785	698	953	1060	1150
	118,639	331	465	554	640	750	833	916	1020	1110
	113,716	312	441	526	809	714	793	873	716	1060
	107.909	289	412	493	570	671	746	821	920	995
	99.154	253	365	439	511	603	672	741	832	901
	83.570	214	309	372	432	510	568	929	703	761
	75.600*	197	283	339	394	464	517	570	629	691
	64.219	168	241	289	335	395	440	485	544	588
	58.139	147	212	256	298	352	393	433	486	527
	51.828	132	191	230	267	316	352	388	435	471
	40.741	114	160	191	220	258	287	315	352	381
	25.433	83.5	112	131	150	173	191	209	232	250
	14.432	55.3	71.6	82.5	92.9	106	116	126	140	150
	based on hourly and daily rain	ily rainfall	data	at the Royal		Observatory, 1884-1939	4-1939	1947-1980	980)	

EXTREME INTENSITY OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS

	PARAM	PARAMETERS				RETURN PE	return periods (years)	ARS)			
DURATION	=	1/R	2	5	10	20	50	100	200	500	1000
	mm h-1	mm h-1	m h-1	mm h ⁻¹	mm h-1	mm h ⁻¹	mm h-1	mm h ⁻¹	mm h-1	mm h-1	mm h_1
120 min.	41.2	14.415	46.5	62.8	73.6	84.0	97.4	108	118	131	141
. 09	61.8	18,658	9.89	89.8	104	117	135	148	161	178	191
30 "	81.7	23,500	90.3	117	135	151	173	190	506	228	244
15 =	103.5	26.500	113	143	163	182	207	225	244	268	287
10,	115.0*	29,000*	126	158	180	201	228	248	269	295	315
ر ت	137.0*	32.500*	149	186	210	234	264	287	309	339	361
. 2	167.0*	\$7.000*	181	222	250	277	311	337	363	397	423
60 seconds	189.0*	40.000*	204	249	279	308	345	373	401	438	465
30 "	220.0*	43.500*	236	285	518	349	390	420	450	490	520
15 =	226.4	45.805	243	295	529	362	405	437	469	511	543
(based on	tilting sip	based on tilting siphon records,	, 1947-1980	30 (Royal C	Observatory)	and	Jardi records	ls at Ki	ng's Par	at King's Park, 1952-1980	1980)

* interpolated data

11

TABLE III. EXTREME DEPTH OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS

(JENKINSON'S 1955 SOLUTION : SMOOTHING OF PARAMETERS INCLUDED) $X = X_0 + cx \left(\frac{1-\frac{e}{k}}{k}\right)$; when k = 0 $X = X_0 + cx$

1		T															_
1000	盟	1370	1220	1100	1080	1030	1010	926	779	269	677	889	565	394	222	153	
500	шш	1320	1160	1020	166	950	928	870	709	629	605	602	496	359	209	126	
200	um	1250	1070	918	887	845	820	092	621	545	518	501	415	314	192	116	; 1947-1980)
100	mm	1200	1000	839	805	992	740	681	557	485	457	453	360	282	178	109	194 : 194
50	TOTAL	1130	928	160	723	688	662	603	495	428	400	372	311	251	164	101	, 1884-1939
20	mm	1040	823	654	617	586	559	504	415	355	330	299	253	211	144	0.06	Observatory
10	am a	952	737	573	535	508	482	431	356	302	280	250	213	182	128	81.1	Royal
5	mu .	854	642	488	452	428	403	357	297	250	232	204	176	153	111	71.3	data at the
2	mm	683	489	431	328	310	288	251	211	176	165	143	128	112	84.0	55.8	rainfall d
ĸ		0.15	0.07	00.00	-0.02	-0.02	-0.03	-0.05	-0.05	<u>-0.07</u>	-0.10	-0.15	-0.15	90.0-	0.05	90.0	daily
B		173.03	143.81	113.29	\sim	ľ	99.10	89.53	72.00	61.20*	53.43	46.40	37.20	34.44	25.12	14.53	on hourly and
×	>	621.5	437.2	317.9	289.1	271.8	251.3		184.6	153.0*	145.2	125.7	113.7	99.4	74.9	50.5	(pased)
DURATION		31 days		: _	:	: 4		2	24 hours	18 #	12 "	<u>=</u>	: 9	. 7	. 2	1	
	x, x, 10 20 50 100 200 500 700 700 700 700 700 700 700 700 7	x _o	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1320 1160 1160 1160 1250 1320 13	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12 317.9 115.29 0.00 431 488 573 654 760 839 918 1020 11 289.1 107.02 -0.02 328 452 535 617 723 805 887 997 10	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12 289.1 107.02 -0.02 328 452 535 617 723 805 887 997 10 271.8 102.55 -0.02 310 428 508 586 688 766 845 950 10	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12 217.9 113.29 0.00 431 488 573 654 760 839 918 1020 11 289.1 107.02 2.0.02 328 452 535 617 723 805 887 997 10 271.8 102.55 -0.02 310 428 508 586 688 766 845 950 10 251.3 99.10 -0.03 288 403 482 559 662 740 820 928 10	Xo k 2 5 10 20 50 100 200 500 10 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12 317.9 113.29 0.00 431 488 573 654 760 839 918 1020 11 289.1 107.02 328 452 535 617 723 805 887 997 10 271.8 102.55 -0.02 310 428 508 586 688 766 845 950 10 251.3 99.10 -0.03 288 403 482 559 662 740 820 928 10 217.6 89.55 -0.05 251 357 431 504	RATION Xo k 2 5 10 20 50 100 200 500 10 days 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 abys 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 13 abys 113.29 0.00 431 488 573 654 760 839 918 1020 11 289.1 107.02 -0.02 328 452 535 617 723 805 887 997 10 " 271.8 102.55 -0.02 310 428 508 586 688 766 845 950 10 " 251.3 99.10 -0.03 288 403 482 559 662 740 870 980 10 " 2217.6	RATION Xo k 2 5 10 20 50 10 Gays 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 asys 621.5 173.03 0.015 683 854 952 1040 1130 1200 1250 1320 13 asys 621.5 173.29 0.00 431 488 573 654 760 839 918 1020 11 289.1 107.02 -0.02 328 452 535 617 723 805 887 997 10 102.55 -0.02 310 428 508 586 688 766 845 950 10 102.55 -0.02 310 428 559 662 740 820 928 10 184.6 72.00 -0.05 211 297 356 415	IAPTION Xo k 2 5 10 20 50 10 20 50 10 days 621.5 175.03 0.15 683 854 952 1040 1130 1200 1720 13 1 437.2 143.81 0.07 489 642 737 823 928 1000 1750 13 1 289.1 107.02 -0.02 328 442 737 654 760 893 918 1020 11 1 289.1 107.02 -0.02 328 452 535 617 723 805 887 997 10 1 271.8 102.55 -0.02 328 403 482 559 662 740 897 10 1 271.6 89.53 -0.02 288 403 482 559 662 740 805 805 805 805 805 805 <	IAATION Xo k 2 5 10 20 50 100 200 500 10 days 621.5 173.03 0.15 683 654 952 1040 1130 1200 1250 13 " 477.2 147.81 0.07 489 642 737 823 928 1000 1070 1150 12 " 289.1 107.02 -0.02 328 42 757 654 760 839 918 1020 11 " 289.1 107.02 -0.02 328 452 535 617 725 805 887 997 10 " 271.8 102.55 -0.02 310 428 559 662 740 820 99 10 " 251.2 99.10 -0.03 288 403 482 559 662 740 820 928 10 "	Adays 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 12 13.7.9 113.2.9 0.07 489 642 737 823 928 1000 1070 1160 12 12 13.2.9 113.2.5 0.00 431 488 573 654 760 839 918 1020 111 107.02 -0.02 328 452 535 617 723 805 887 997 10 10 102.55 -0.05 211 297 356 662 740 820 928 10 10 10 10 10 10 10 10 10 10 10 10 10	Invertion Xo k 2 5 10 20 50 10 200 500 10 days 621.5 177.03 0.15 683 854 952 1040 1130 1200 1320 1320 13 " 437.2 143.81 0.07 489 642 737 823 928 1000 1070 1160 12 " 277.2 143.81 0.00 431 488 573 654 760 839 918 1020 1160 12 " 277.8 102.55 -0.02 328 452 535 654 760 887 997 10 " 277.8 102.55 -0.02 328 408 559 662 740 820 950 10 " 251.3 99.10 -0.05 251 357 441 504 603 662 740 805 895 10	Invertion Xo k 2 5 10 20 50 10 days 621.5 173.03 0.15 683 654 952 1040 1130 1200 1250 130 13 17.29 0.07 489 642 737 823 928 1000 1070 1160 120 1 289.1 0.07 489 642 737 823 928 1000 1070 1160 120 1 289.1 0.07 489 642 737 823 928 1000 1070 1160 120 1 289.1 0.07 489 642 737 654 654 760 887 997 10 1 27.0 0.02 328 452 535 617 760 887 997 10 1 27.6 89.53 -0.02 271 428 508 668 766	Adays 621.5 k 2 5 10 20 50 10 days 621.5 173.03 0.15 683 854 952 1040 1130 1200 1250 1320 13 " 437.2 143.81 0.07 431 489 642 737 823 928 1000 1070 150 15 15 10 10 10 10 10 10 10 11 10 <th< td=""></th<>

EXTREME INTENSITY OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS

DURATION X						RETURN	PERIOD (Y	(YEARS)			
-	`	עג	2	5	10	20	50	100	200	500	1000
	\$		mm h_1	mm h-1	mm h-1	mm h_1	L_d mm	m h-1	mm h-1	rm h-1	h mm
120 min. 40.0	14.00	0.10	45.0	59.5	68.2	0.97	85.2	91.6	97.6	105	110
0.69 " 63.0	18.69	0.12	69.7	88.7	99.9	110	121	129	136	145	151
30 " 82.8	24.00	0.10	91.4	116	131	144	160	171	181	194	203
15 " 105.4		0.20	116	143	158	170	184	193	200	209	214
119.0*		0.23	130	160	175	188	201	210	217	225	230
5 " 140.0*		0.27	153	186	202	216	229	237	244	251	256
2 " 172.0*	* 43.00*	0.32	187	223	241	254	268	276	282	288	292
60 seconds 195.0*	* 47.00*	0.35	211	250	268	282	295	302	308	314	317
30 " 21 6. 0*	* 50.00*	0.37	233	274	292	306	319	326	332	358	341
15 " 235.8	53.47	0.40	254	596	315	329	341	348	353	358	361

* interpolated data

TABLE IV. EXTREME DEPTH OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS

(JENKINSON'S MAXIMUM LIKELIHOOD METHOD : SMOOTHING OF PARAMETERS INCLUDED) $X = X_0 + \alpha(\frac{1-e^{-kY}}{k})$; when k = 0 $X = X_0 + \alpha Y$

		PARAMETERS					RETURN	PERIOD (YEARS)	ARS)			
DURATION	×	8	¥	2	5	10	20	50	100	200	500	1000
	0				mm m	mm					8	1
days	620.1	173.40	0.14	682	855	955	1040	1140	1210	1270	1340	1390
.=	437.4	145.27	0.07	490	644	740	827	933	1010	1080	1170	1230
=	317.9	113.75	0.01	359	487	571	651	753	829	905	1000	1080
=	288.9	107.45	-0.02	328	453	536	618	725	807	688	1000	1080
=	271.3	102,22	-0.04	309	429	512	594	703	788	874	992	1080
=	251.5	99.72	-0.04	288	406	486	995	673	755	840	955	1050
=	217.3	88.76	-0.05	250	356	429	502	009	929	756	864	950
hours	184.3	67.20	-0.07	209	291	348	406	486	549	615	707	781
=	167.4*	61.20*	-0.10	190	566	322	379	459	525	595	:695	776
=	144.3	52.05	-0.12	164	230	279	330	403	464	529	625	704
=	125.2	41.94	-0.17	141	197	240	287	357	418	485	588	677
=	113.8	38.52	-0.15	128	179	217	258	318	369	425	509	581
=	99.1	32.00	-0.08	111	150	178	506	246	277	310	357	394
=	74.4	24.25	0.05	83.2	109	126	141	160	174	187	204	216
**	50.3	14.10	0.10	55.4	6.69	78.7	86.5	95.9	102	108	116	121
	(based o	on hourly and	nd daily rainfall	١.	data at the	Royal	Observatory	, 1884-1939	9 ; 1947–1980	.1980)		
					***************************************					***************************************		

EXTREME INTENSITY OF RAINFALL CORRESPONDING TO VARIOUS RETURN PERIODS

	500 1000	mm h-1 mm h-1	14 120		76 182	205 210		54 258		319 322	339 342		980)
	200 5	mm h ⁻¹ mm	104					246 2		-			Park, 1952-1980
ars)	100	m h_1 m	97.1		160			238				339	King's Par
RETURN PERIOD (YEARS)	50	h_1	89.5	118	151	181	202	230	269	300	323	334	records at
RETURN	20	mm h_1	78.9	106	158	168	187	215	255	287	310	324	and Jardi
	10	- 나 때	70.4	96.3	127	156	174	202	241	273	297	313	ervatory)
:	5	m h_1	61.2	85.1	114	142	158	185	223	255	279	596	Royal Observatory)
	2	m h-1	46.6	66.5	91.4	115	129	153	187	216	238	256	
	ذ .	4	0.05	0.10	0.15	0.20	0.20	0.25	0.30	0.35	0.40	0.48	ords, 194
PARAMETERS	`	ረ ረ	13.48	18.00	23.00	28.00	31.00*	36.00*	45.00*	44.00*	52.00*	54.40	siphon records, 1947-1980
	>	°	41.7	0.09	83.2	105.2	118.0*	140.0*	172.0*	200.0*	220.0*	237.8	based on tilting
<u>1</u>	DURATION		20 min.	. 09	30 "	15 =	10 :	= 5	2 =	60 seconds	30 "	15 "	(based

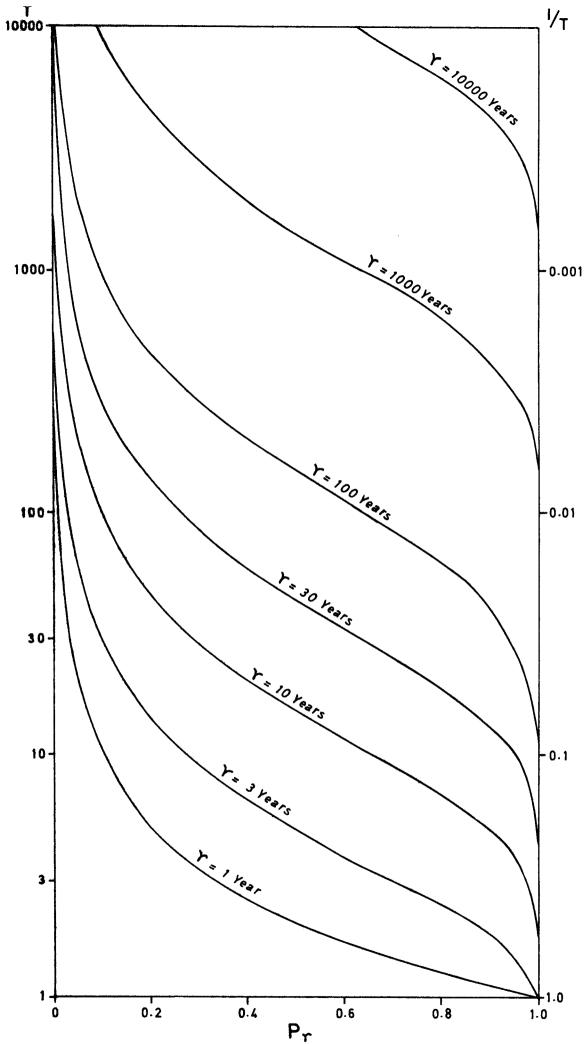
* interpolated data

CALCULATED VALUES OF A, b AND c AS A FUNCTION OF THE RETURN PERIOD, UNDER DIFFERENT EXTREME-VALUE DISTRIBUTIONS TABLE V.

Return		Gumbe1		Jen	Jenkinson's 1955 solution	, 1955 m	Jenki	Jerkingon's maximum likelihood	maximum ood
period	ď	,a	O	ત્ય	۵	O	ರ	р	O
2 yr	511	4.7	0.51	586	5.4	0.54	545	4.7	0.52
5	57.1	4.5	0.47	661	5.6	0.50	611	4.8	0.49
10	622	4.5	0.45	674	5.6	0.48	624	4.8	0.46
20	682	4.5	0.44	699	5.5	0.45	627	4.8	0.44
50	748	4.5	0.43	648	5.3	0.42	209	4.6	0.40
100	789	4.5	0.42	633	5.2	0.39	585	4.4	0.38
200	840	4.5	0.42	599	4.8	0.37	558	4.1	0.35
500	918	4.6	0.41	593	4.4	0.33	513	3.5	0.31
1000	896	4.6	0.41	534	4.0	0.31	485	3.1	0.29

TABLE VI. MEAN PERCENTAGE DEVIATION OF RAINFALL INTENSITY VALUES FROM FITTED CURVES AS A FUNCTION OF DURATION. POSITIVE VALUES MEAN THAT THE INTENSITY VALUES ARE GREATER THAN THOSE WORKED OUT FROM THE FITTED GRAPHS

Mean percentage deviation Duration (min)	Gumbel	Jenkinson's 1955 solution	Jenkinson's maximum likelihood
240	- 9.2	- 4.3	-6. 7
120	3.8	- 2 . 1	3. 2
60	7.9	6.4	4.7
30	6.1	8.2	4•3
15	-0.5	- 0•5	-0.3
10	- 3.6	- 3.3	-1.6
5	-7.4	-7. 2	-5. 8
2	-7.0	- 5•9	-5•4
1	- 3.6	-2.2	-0.9
0.5	4.6	2.4	2.6
0.25	6.1	7.1	4.3



The probability that a level with a return period 'T' will be reached or exceeded at least once in a period 'r'
16 Figure 1.

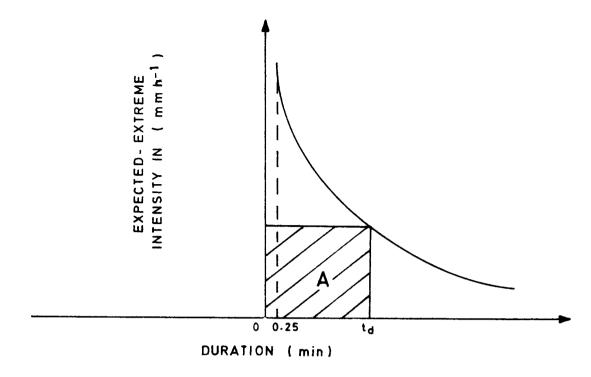


Figure 2. A diagram showing a typical plot of expected extreme intensity versus duration for a given return period

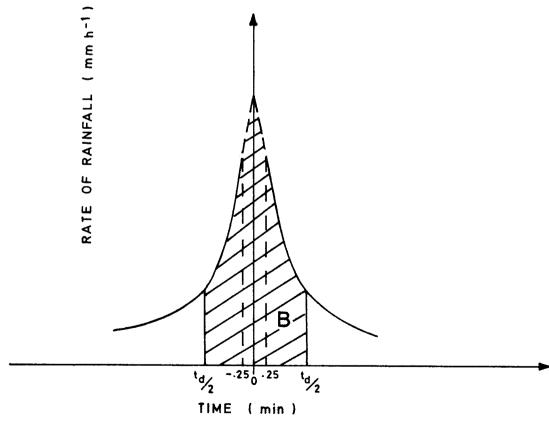


Figure 3. A storm profile, corresponding to the same return period, constructed under the constraint that area A = area B for any value of $t_{\rm d}$

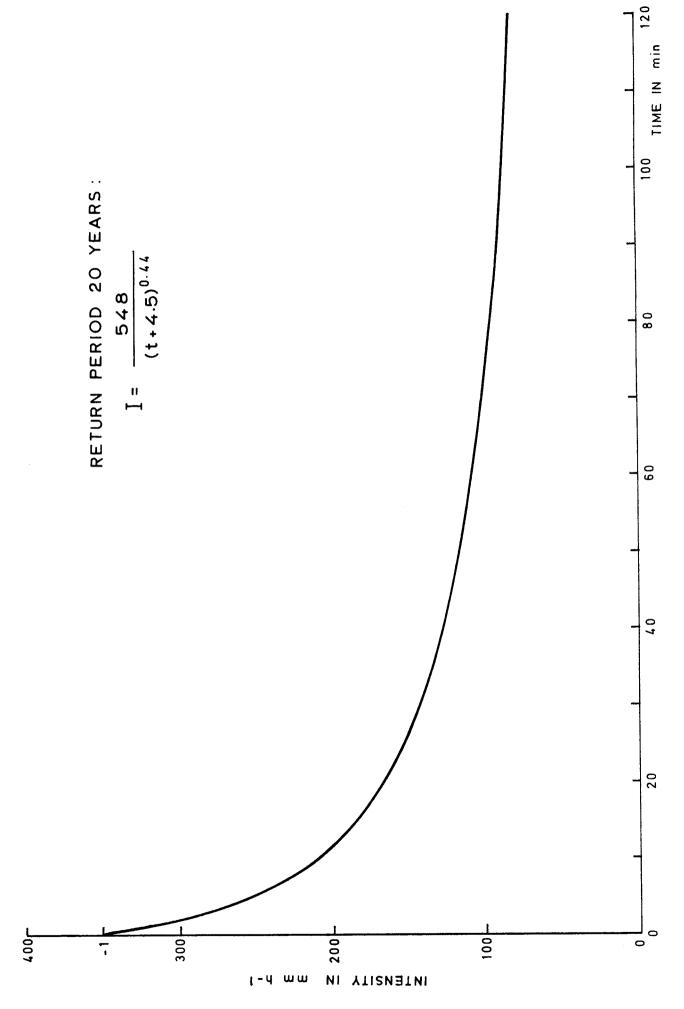


Figure 4. Rainfall intensity versus duration for a return period of 20 years (Gumbel's solution)

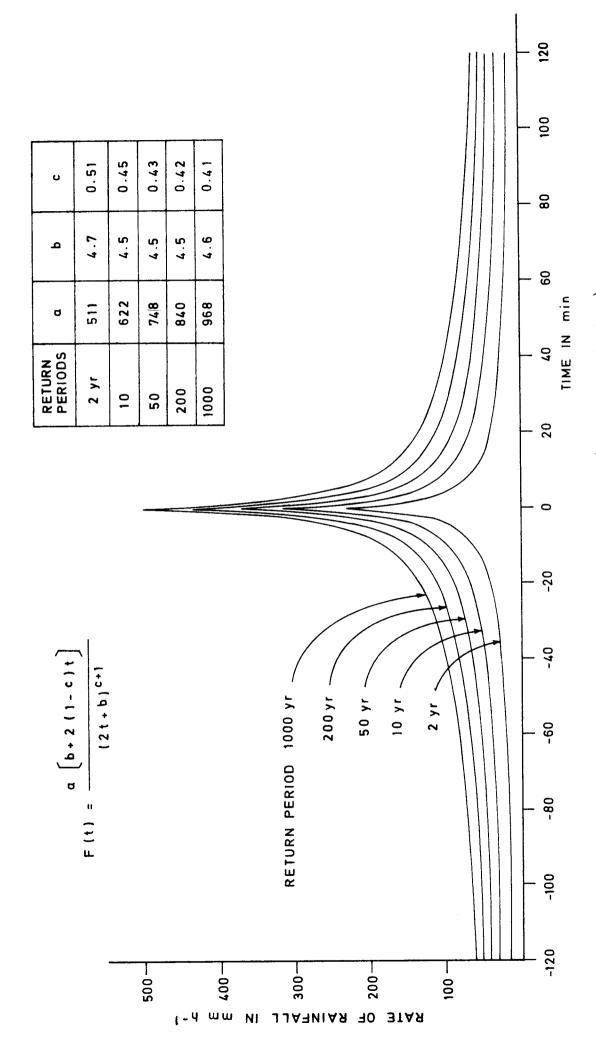


Figure 5. Storm profiles for various return periods (Gumbel's solution)

Storm profiles for various return periods (Jenkinson's 1955 solution) Figure 6.

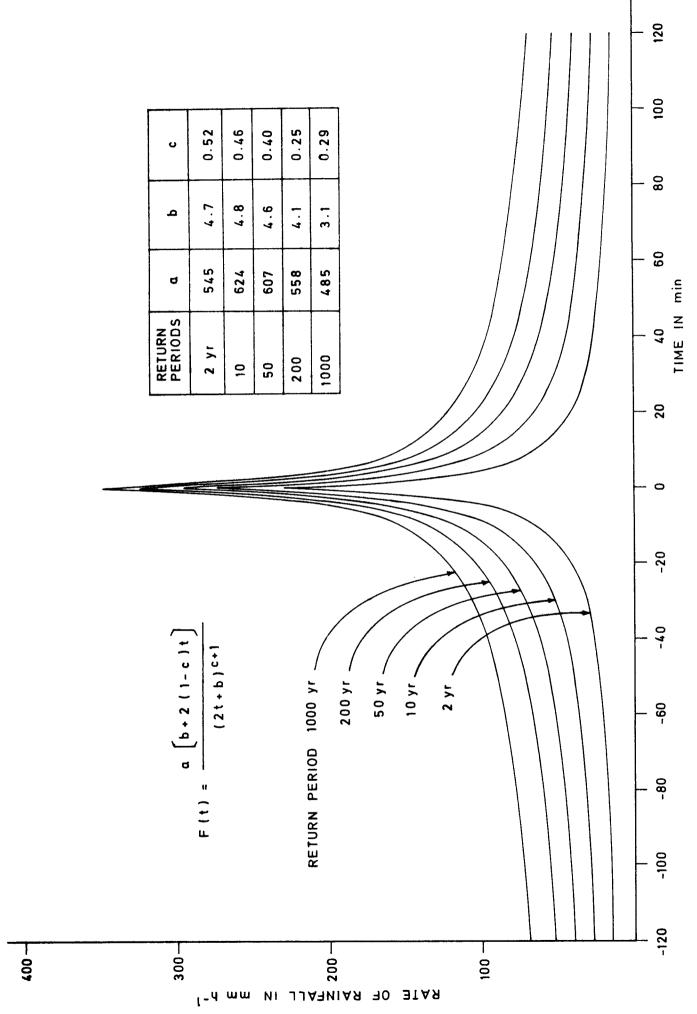


Figure 7. Storm profiles for various return periods (Jenkinson's maximum likelihood solution)

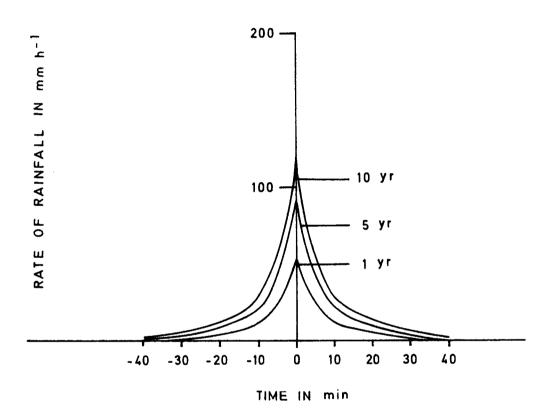
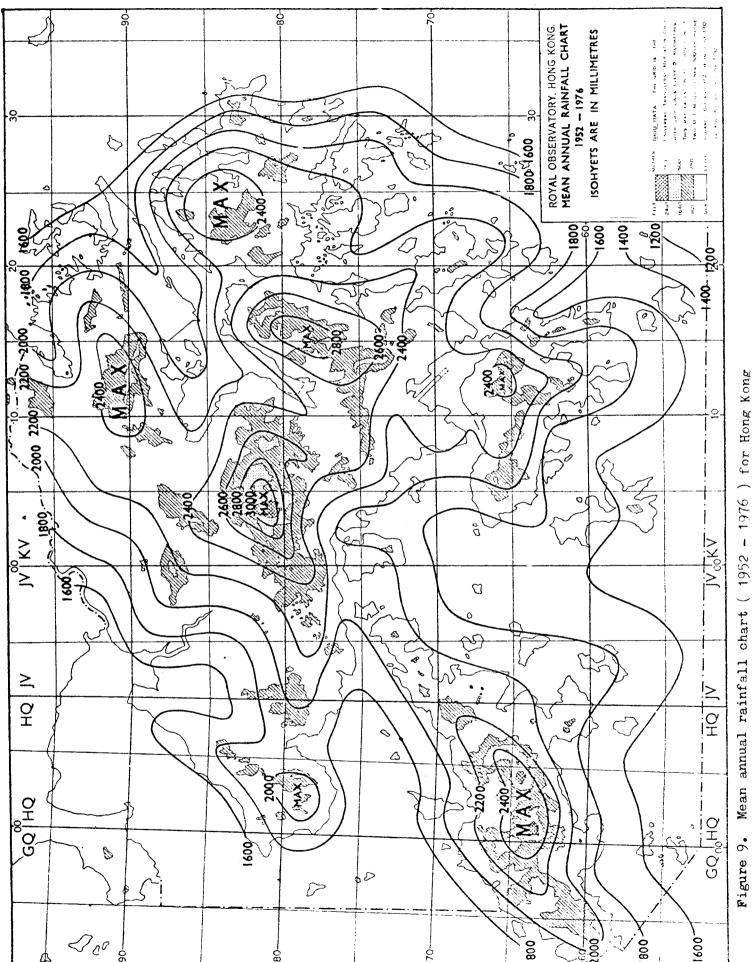


Figure 8. Storm profiles for U.K. (Road Note No. 35)



APPENDIX I

A LIST OF RECENT RAINSTORMS REPORTED TO HAVE CAUSED DAMAGE IN HONG KONG

Date	Rainfall +	Effect*	Cause
9/6/60	236.1 mm in 1 day	45 deaths; 11 people missing; 127 injured; 15 000 affected	Typhoon Mary
26/5/62	114.2 mm in 1 day	Some damage in Sai Kung	Passage of a trough from south
1/9/62	203.0 mm in 1 day	130 deaths; 53 missing; 72 000 homeless	Typhoon Wanda
28/5/64	248.5 mm in 1 day	Flooding of low-lying paddi fields	Typhoon Viola
10-11/9/64	177.0 mm in 2 days	Numerous landslides	Typhoon Sally
12-13/10/64	331.1 mm in 2 days	26 deaths; numerous landslides	Typhoon Dot
26/9/65 to 1/10/65	534.2 mm in 6 days	6 deaths; 200 homeless; widespread flooding; numerous minor landslides	T.S. Agnes
4/4/66	190.2 mm in 1 day	15 deaths	Thunderstorms
12/6/66	382.6 mm in 1 day; 108.2 mm in 1 h; 157.0 mm in 1 h; at Aberdeen is highest on record; monthly rainfall 962.9 mm highest since May 1889	64 deaths; 29 injured; disastrous landslides and washouts including Peak Road, Stubbs Road and Ming Yuen St.; 8 561 evacuated from their homes	Synoptic situation and the prolonged period of rain from 2nd to 15th are described in R.O. Occasional Paper No. 7
13-14/7/66	160.0 mm in 2 days	1 death, several injured	S.T.S. Lola
18/8/66	125.9 mm in 1 day	Short period of flooding in low-lying areas	SW Monsoon as T.S. Tess landed near Fuzhou
13-14/7/67	126.0 mm in 2 days, more than 250.0 mm fell near Plover Cove and Tai Lam Chung	no casulties	Upper air disturbance
20 - 22/8/67	191.8 mm in 3 days	3 injured; 1 500 evacuated from their homes	T. Kate following a T.D. which caused 177.1 mm between 10-14 Aug and T.S. Iris which caused 135.0 mm between 15 and 17 Aug
		24	•

APPENDIX I (cont'd)

Date	Rainfall ⁺	Effect#	Cause
12=13/6/68	326.2 mm in 2 days	22 deaths; 7 injured; 10 landslides	A surface trough
20*22/8/68	257.9 mm in 3 days	3 cases of landslides;4 injured;5 000 evacuated from their homes	Typhoon Shirley
10-11/8/69	220.8 mm on 11 Aug and 292.6 mm on the 2 days after the typhoon had landed near Fughou	Urban flooding disrupted traffic; 2 000 evacuated in Shatin; landslides occurred over many hilly areas	Typhoon Betty described in Royal Observatory Tech. Note No. 40
13/5/70	265.1 mm in 1 day and 139.6 mm in 2 h	Numerous landslides and floods	A trough
16/7/70	378 mm h ⁻¹ -highest instantaneous rate on record at R.O.	landslides reported	T.S. Ruby
2-3/8/70	223.5 mm in 2 days	2 killed by lightning; 4 huts affected by flooding	T.D.
16/9/70	197.5 mm in 24 hours at Taipo (R.O. rainfall was only 2.2 mm/day)	368 evacuated from their homes in Taipo	Remnant of T. Georgia which landed on 14 September.
19/5/71	106.1 mm in 1 day	Flooding in low-lying area; 29 people homeless	Onset of the Southwest Monsoon
1 8/6/ 71	95.3 mm in 1 day; 183.2 mm in 2 days 17-18 June	2 dead; 30 injured, 100 affected by flooding in N.T.	Typhoon Freda
22/7/71	142.5 mm in 1 day	38 injured; flooding in the N.T.	Typhoon Lucy
17/8/71	288.1 mm in 1 day; an instantaneous rate of 513.0 mm h ⁻¹ was recorded at Tate's Cairn at around 8.30 a.m.	100 killed; hundreds injured	Typhoon Rose
18/ 1 2/71	97.8 mm in 1 day	Some flooding in N.T.; one landslide	Upper air disturbance
7-11/5/72	325.0 mm in 2 days; 444.4 mm in 5 days	Flooding reported in 8 areas; 2 landslides; 6 055 people homeless	A trough.

APPENDIX I (cont'd)

Date	Rainfall*	Effect*	Cause
16-18/6/72	652.3 mm in 3 days; 232.6 mm on the 18th including 98.7 mm from 11 a.m. to noon	Disastrous landslides at Sau Mau Ping and Po Shan Road; 53 cases of flooding; 138 killed; 56 injured; 7 800 homeless	Trough of low pressure described in Royal Observatory Tech. Note No. 51
1 9–2 1/8 /7 2	186.8 mm on the 20th; 288.4 mm in 3 days	1 house collapsed; 9 wooden huts damaged; 57 people affected	Typhoon Betty landed near Fuzhou on 19 Aug described in Royal Observatory Tech. Note No. 40
2 7/6/7 3	128.9 mm in 1 day		A trough
16-17/7/73	214.3 mm in 2 days; 172.1 mm on 17 July	Flooding with water 1 m depth in Tin Shui Wai, Yuen Long; 238 people affected	Typhoon Dot
9-13/8/73	247.8 mm in 5 days	1 person killed	Typhoon Georgia
21/8/73	212 mm in 5 hours (5 a.m 10 a.m.); 251.5 mm in 1 day	<pre>3 cases of flooding; 1 killed; 1 injured</pre>	Tropical Storm Joan
19/ 9 /73	83.2 mm in 1 day (Note: 1973 was the wettest year in H.K. with 3 100.4 mm	Flooding with depth of 1 m in Sai Kung; 10 affected	A cold front
7-9/4/74	142.9 mm in 3 days	Rock falls on roads in N.T.	A cold front
23/8/74	128.3 mm in 1 day	Flooding in urban areas	Triggered by S.T.S. Mary
18 -2 0/10/74	459.5 mm in 3 days	Flooding in low-lying areas; 1 000 people evacuated from their homes	Typhoon Carmen; the wettest October typhoon on record
29-31/10/74	225.2 mm in 3 days	Some flooding and minor landslides	Typhoon Elaine
2/ 1 2/74	177.3 mm in 1 day highest on record in December	No casulties	T. Irma
2 8–3 0/4/75	149.7 mm in 1 day; 92.4 mm from noon to 1 p.m. on 30 April; 304.9 mm in 3 days	Widespread flooding, esp. in Sai Kung and Kowloon Bay	A trough
20/5/7 5	215.7 mm in 1 day	Flooding in Kwai Chung, Sai Kung, Tai Po, Kwun Tong; flooding with water 1 m high in Pokfulam Village	A trough

APPENDIX I (cont'd)

Date	Rainfall ⁺	Effect*	Cause
1-7/6/75	82.1 mm on 5 Jun 271.1 mm in 7 days	Widespread flooding	SW monsoon
12 –1 6/7/75	240.7 mm in 5 days	2 drowned in N.T.	Upper air disturbances
2 -3/ 6/76	197.4 mm in 2 days	Flooding in Sam Shing Hui and Tuen Mun with water 8 ft high; 200 people evacuated from their homes	A trough
2 4-2 7/ 7 / 7 6	271.7 mm in 4 days	Some minor flooding	S.T.S. Violet
2 4-2 5/8/ 7 6	416.2 mm in 24 hours starting 11 a.m. on 24 Aug; 511.6 mm in 2 days	Landslips occurred in many places including Sau Mau Ping where 18 people were killed; 2 424 people evacuated from their homes; water rose to 4 ft on King's Road, H.K. Island	S.T.S. Ellen
23/6/77	91.9 mm in 1 day	60 people homeless; places affected including Lam Tin, Lei U Mun and Sau Mau Ping	A trough
4 6 /9/77	267.6 mm was recorded over the weekend	Many roads flooded. A wooden hut was damaged near Lion Rock by a large boulder	T.S. Carla
2/6/78	97.6 mm in 1 day; 70 mm in 1 hour	Flooding in Chai Wan and Shatin	A trough
2 4-3 0/7/78	502.4 mm in 7 days	2 drowned and 1 killed in a landslip	S.T.S. Agnes
16-17/10/78	120.9 mm on 16 October and 284.8 mm on 17 October	Minor landslips and floodings	S.T.S. Nina
11/6/79	113.6 mm in 1 day	3 cases of landslips; 200 people are evacuated from their homes	A trough
2 9-3 1/7/79	260.2 mm in 3 days	Flooding in the New Territories	S.T.S. Gordon
2/ 8/7 9	209.0 mm in 1 day	Severe damage to property in H.K.; numerous minor landslips occurred	Typhoon Hope

APPENDIX I (contid)

Date	Rainfall ⁺	Effect*	Cause
25/9/79	245.2 mm in 1 day	Many roads fixeded; several minor landslips occurred	Typhoon Mac
24 ~2 8/6/80	82.7 mm in 5 days	Some roads flooded; a few minor landslips	S.T.S. Herbert
10+14/7/80	190.8 mm in 4 days	1 missing at Sheung Shui in N.T.	S.T.S. Ida
22-23/7/80	61.9 mm in 2 days	2 killed, 1 missing and 59 injured	Typhoon Joe
26-28/7/80	136.3 mm in 3 days	2 landslips; flooding at Tai Po and Sheung Shui	Typhoon Kim
10-11/5/81	178.8 mm in 2 days	Widespread landslips; flooding in Sha Tin and Tai Po; 1 killed and 650 evacuated	A trough
30-31 /5/81	88.4 mm in 2 days	Landslips in Yau Tong, Kwun Tong and Sau Mau Ping; Over 100 people evacuated	A trough
5/6/81	33.3 mm in 1 day	Landslip on King's Road, 1 killed	A trough
5 -8/7 /81	83.7 mm in 4 days	Landslips at Diamond Hill, Yau Tong and Shau Kei Wan; 32 injured	S.T.S. Lynn
2-3/8/81	5.7 mm in 2 days	Landslip at Sau Mau Ping	A trough
4/9/81	126.1 mm in 1 day	Landslips at Sau Mau Ping, Kwun Tong and Diamond Hill	A trough

^{*} All rainfall amounts are measured at the Royal Observatory unless otherwise stated.

^{*} Effects of the rainstorms are quoted from ESCAP Reports (1960-1980) prepared by the Royal Observatory.

APPENDIX II

DEFERMINATION OF CONSTANTS a, b AND c IN EQUATION 5

The expression for I is:

$$I = \frac{a}{(t+b)^c}$$

Taking logarithms on both sides,

$$log I = -c log (t + b) + log a.$$

Provided the value of b is known, a linear regression of log I versus log (t + b) will give -c as the slope and log a as the y - intercept.

With the data set $\{t_i, I_i : i = 1, ..., n\}$ given, various values of b were used, by trial and error, until a value of b was found to give the least sum of squares:

$$\sum_{i=1}^{n} \left(\log I_{i} + c \log (t_{i} + b) - \log a \right)^{2}.$$

This value of b was thus adopted and then used in the standard procedures of least-square fit in the graph of $\log I$ versus $\log (t + b)$ to give estimates of a and c.

APPENDIX III

DERIVATION OF AN EXPRESSION FOR A STORM PROFILE FROM THE EXTREME RATE OF RAINFALL VERSUS DURATION FORMULA

Since the profile is assumed to be symmetric with the maximum occurring at t = 0, only the formula for $t \ge 0$ is derived. The formula for t < 0 can be found similarly.

Let the shaded area in figure 2 be A and that in figure 3 be B and the profile be f(t).

$$A = \frac{at}{(t+b)^{c}}$$

$$E = 2 \times \int_{0}^{t/2} f(t) dt$$

Equating the two areas:

$$2 \times \int_{0}^{t/2} f(t) dt = \frac{at}{(t+b)^{c}}$$

$$\int_{0}^{t/2} f(t) dt = \frac{at}{2(t+b)^{c}}$$

However, the left hand side = F(t/2) - F(o)

if
$$\frac{dF(t)}{dt} = f(t)$$
.

..
$$F(t/2) - F(0) = \frac{at}{2(t+b)^{c}}$$

Differentiate both sides with respect to t:

$$\frac{\mathrm{d}\mathbf{F}(t/2)}{\mathrm{d}(t/2)} \cdot \frac{\mathrm{d}(t/2)}{\mathrm{d}t} = \frac{\mathrm{a}\left[b+(1-c)t\right]}{2(t+b)^{c+1}}$$

Differentiate both sides with respect to t:

$$\frac{\mathrm{d}F(t/2)}{\mathrm{d}(t/2)} \cdot \frac{\mathrm{d}(t/2)}{\mathrm{d}t} = \frac{\mathrm{a}\left[b+(1-c)t\right]}{2(t+b)^{c+1}}$$

Let
$$t = t/2$$
, then

$$\frac{\mathrm{d}\mathbf{F}(t)}{\mathrm{d}t} = \frac{\mathrm{a}\left[\mathrm{b}+2(1-\mathrm{c})t\right]}{(2t+\mathrm{b})^{\mathrm{c}+1}}$$

But
$$\frac{dF(t)}{dt} = f(t)$$

:
$$f(t) = \frac{a[b+2(1-c)t]}{(2t+b)^{c+1}}$$