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Trapped Lee Waves over Lantau Island: Case Studies

Abstract

The occurrence of both a Foehn effect and trapped lee waves over Lantau Island in Hong Kong have been identified using meteorological observations made simultaneously on both sides of the Island. Three typical cases are presented in this paper, with winds coming from the north in two instances and from the opposite southeasterly direction in the third. Characteristic increase of air temperature and drop in relative humidity associated with the phenomenon were observed on the mountain lee in all three cases. The calculated wavelengths of trapped lee waves from most radiosonde ascents were very similar to the half-width of the ridge.

Introduction

Meteorological observations from weather stations located on either side of the mountains on Lantau Island provided an unique opportunity for studying the Foehn phenomenon in Hong Kong. Although the ridge involved was less than 1,000 metres high, various characteristics of Foehn winds have been observed.

The Foehn effect (Brinkman, 1971) describes a phenomenon in which a moist horizontal airstream impinges almost perpendicularly on a mountain ridge where forced lifting causes condensation and sometimes rain on the windward slope. As the airstream descends the leeside, it becomes warmer and drier as a result of adiabatic heating.

Moreover, in a stably stratified atmosphere and under favourable conditions, the airstream would oscillate vertically to form lee waves and propagate long distances downstream of the ridge. Rotor streaming is often found below the wave crests of reasonable amplitudes (Wallington, 1960). Lee waves are observed downstream of high mountain ridges such as the Alps and the Rockies. There have also been reports of such occurrences in the U.K. (Rogers *et al.*, 1995).

The physical properties of lee waves depend on the intervening mountain barrier, stability of the atmosphere and characteristics of the impinging airstream. Stationary waves occurring downstream of ridges are shown to be trapped lee waves (Smith 1979).

Theory of Trapped Lee Waves

There are a number of reviews on mountain lee waves (Holton 1992 and Smith 1979). In this section, only the major theoretical results relevant to the present analysis will be discussed. Detailed mathematical derivations of these results can be found in the above reviews.

Consider an airstream flowing nearly perpendicularly across a long, isolated ridge throughout a considerable depth. The atmosphere is assumed to be stably stratified at least in the region disturbed by the ridge. In the linear gravity wave theory, the ridge is assumed to induce small perturbations in the airstream about a basic state at rest. Moreover, horizontal length scales of the systems, namely the width of the ridge and the wavelength of the lee waves, are in the range of 100 m to 50 km, that is, small compared with the radius of the Earth so that Coriolis forces are negligible. Using Boussinesq approximation, the vertical velocity field, *w*, of the airstream satisfies:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \frac{N^2}{\overline{u}^2} w = 0. \qquad (1)$$

Here x is the horizontal direction across the ridge, y parallel to the ridge, and z the vertical direction. The parameter $\overline{u}$ refers to the mean horizontal velocity of the airstream impinging on the ridge. *N* denotes the Brunt – Väisälä frequency, namely, $N = \sqrt{(g / \vartheta)(\partial \vartheta / \partial z)}$, where g is the acceleration due to gravity and ϑ the potential temperature. Equation (1) refers to vertically propagating wave motions on the lee side of the ridge, commonly known as lee waves.

If mean velocity of the airstream varies with height, the equation for vertical velocity field will be similar to (1) above, with coefficient of w in the last term on left hand side replaced by:

$$l^2 = \frac{N^2}{\overline{u}^2} - \frac{1}{\overline{u}} \frac{d^2 \overline{u}}{dz^2}. \qquad (2)$$

l is called the Scorer parameter (Scorer, 1949). It plays the same role as $N / \overline{u}$ in equation (1). If wavenumber of the lee wave is smaller than l, the natural frequency of the atmosphere is able to support the vertical oscillatory motion of the perturbed air parcels. Wave motions would then be sustained. On the contrary, if wavenumber of the lee wave is greater than l, the atmosphere no longer supports air parcel oscillations and wave motions will decay with height.

Evaluation of the Scorer parameter as a function of altitude requires the input of the first derivative of potential temperature and the second derivative of mean airstream velocity with height. In some calculations (Smith 1976) the latter is neglected by assuming that the second term in equation (2) should be much smaller in magnitude than the first term. This is equivalent to assuming that the mean airstream velocity is a linear function of altitude.

Now suppose that in the region of airstream perturbed by the ridge, the Scorer parameter decreases with height. For a lee wave with wavenumber k in this range of decreasing l, wave motions will exist in the lower layer of the airstream while no wave motions can be observed in the upper layer. The vertically propagating lee waves in the lower layer are then *'reflected downwards'* when encountering the boundary of lower and upper layers of the atmosphere, and the upward and downward waves superimpose on each other to result in a standing wave. Such standing waves are called trapped lee waves. Their static property explains why many natural lee wave phenomena can persist for a number of hours or even a few days.

For a moist atmosphere, virtual potential temperature ϑ_V should be used instead of potential temperature in considering vertical motion of air parcels (Stull, 1991). By neglecting the second term in equation (2), the following formula is employed throughout this paper for evaluating the Scorer parameter:

$$l^2 = \frac{g\dfrac{d\ln\vartheta_V}{dz}}{\bar{u}^2} \tag{3}$$

In some calculations (Ajit Tyagi and Madan, 1989), the component of horizontal velocity of the airstream perpendicular to the ridge was adopted in equation (3). This gives more reliable estimation of the Scorer parameter provided that horizontal velocities in the upper air are measured under no orographic effects.

Theoretical calculation of wavenumber and wavelength of trapped lee waves requires a good representation of the cross-sectional profile of the perturbation ridge. The resulting equations are in general not analytically solvable. However, the wavelengths may be approximated as follows. Assume that in the region of airstream perturbed by the ridge, the Scorer parameter drops from l_{max} to l_{min}. Then for trapped lee waves to occur, their wavenumber k must satisfy:

$$l_{max} > k > l_{min}\,. \tag{4}$$

The region from l_{max} to k corresponds to the upper layer of atmosphere in which wave motions are prohibited, while the region from k to l_{min} is the lower layer where lee waves are allowed. Since wavenumber k of a wave is related to its wavelength λ by $k = 2\pi/\lambda$, equation (4) gives the range of wavelength of the trapped lee wave:

$$\frac{2\pi}{l_{max}} < \lambda < \frac{2\pi}{l_{min}}. \tag{5}$$

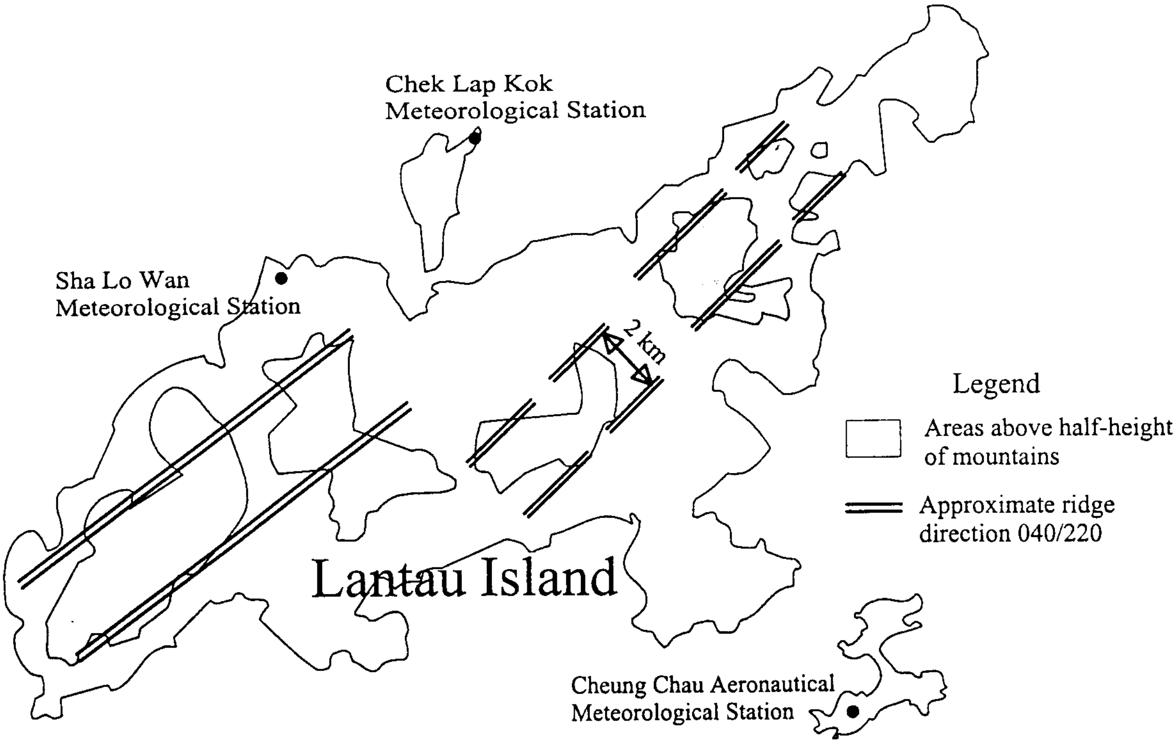
The average of upper and lower bounds for wavelengths in equation (5) can be taken as a scale of the wavelength.

The calculated wavelengths of trapped lee waves are then compared with the width scale of the ridge. According to Hunt's classification (Stull, 1991), when they are comparable in magnitude, there may be resonance in lee wave generation. Amplitude of trapped lee waves will be the largest and strong vertical circulation will be observed beneath the wave crests. However, Smith (1976) has shown that, although the classical theory presented here provides good approximation to the wavelengths, nonlinear effects play significant role in determining the wave amplitudes. In fact, when perturbation from the basic state in the airstream becomes large, the linearizaton process for deducing the wave equations will not be justified.

Trapped Lee Waves over Lantau Island

Lantau is a hilly island located to the southwest of Hong Kong. Figure 1 shows the area on the island which is more than half the maximum height of the ridge. This area of high ground extends roughly from southwest to northeast and has a half-width of about 2 km. In the following analysis orientation of the ridge is taken to be 40 degrees from the north.

Figure 1 Map of Lantau Island showing the location of the weather stations and the estimated half-width of the Lantau ridge.



Weather stations at Cheung Chau, Chek Lap Kok and Sha Lo Wan provided vital surface temperature, humidity and wind data for this investigation. Radiosonde data from the King's Park Meteorological Station were used to evaluate vertical variation of the Scorer parameter, humidity mixing ratio and virtual potential temperatures (Rogers and Yau, 1989). The first derivatives are calculated by using linear interpolations between the actual data points.

Case Studies

(a) Case I: Northerly Winds (28 November to 1 December 1982)

The territory was under the influence of a prevailing northerly wind during the study period. Figure 2 illustrates that temperatures at Cheung Chau, down wind of the Lantau ridge, were higher than at Chek Lap Kok by as much as five degrees but at the same time relative humidities were lower by 10%. Figure 3 shows that winds at Cheung Chau were stronger by about three metres per second. Meteorological readings for Chek Lap Kok and Cheung Chau at some selected times are given in Figure 4.

The observations described above are consistent with the Foehn model where the moist northerly airstream was mechanically uplifted across the Lantau ridge depositing much of its water content on the windward side. For example, on 28 November 22.8 and 6.2 millimetres were recorded at Chek Lap Kok and Cheung Chau respectively. As the airstream descended the lee towards Cheung Chau, it warmed up adiabatically and turned dry. This Foehn episode persisted for three days.

Using equation (2), the Scorer parameter profiles were prepared from local upper-air ascents during the time period. Figure 5 shows the selected profile at 8 a.m. of 29 November and a peak can be observed below 1 km, slightly higher than the height of the Lantau ridge. From equation (5), the wavelength of the trapped wave was estimated to be about 1.9 km which was similar to the half-width of the Lantau ridge. These were factors favourable for the formation of trapped lee waves. Table 1 shows the time variation of the estimated wavelengths of trapped waves as deduced from the peak Scorer parameter values.

From surface observations taken at Cheung Chau, it can be deduced that the Foehn ceased for a short while at around 2:00 p.m. on 29 November before the temperatures and relative humidities differential recurred. The wavelength computed from the ascent at 8:00 p.m. on 29 November was about 2.3 km. Peak values of the Scorer parameter were found between 800 and 1,800 m. The phenomenon ceased at around noon of 1 December.

Table 1 The calculated wavelength of trapped lee waves for different soundings in Case I.

Sounding		Calculated wavelength of trapped lee waves (km)
Day	Hour	
29 November	8:00 a.m.	1.9
	2:00 p.m.	2.3
	8:00 p.m.	2.3
30 November	2:00 a.m.	1.8
	8:00 a.m.	2.4
	2:00 p.m.	1.9
	8:00 p.m.	2.0

Figure 2 Case I: Time series of temperature and relative humidity at Cheung Chau and Chek Lap Kok from 28 November to 1 December 1982.

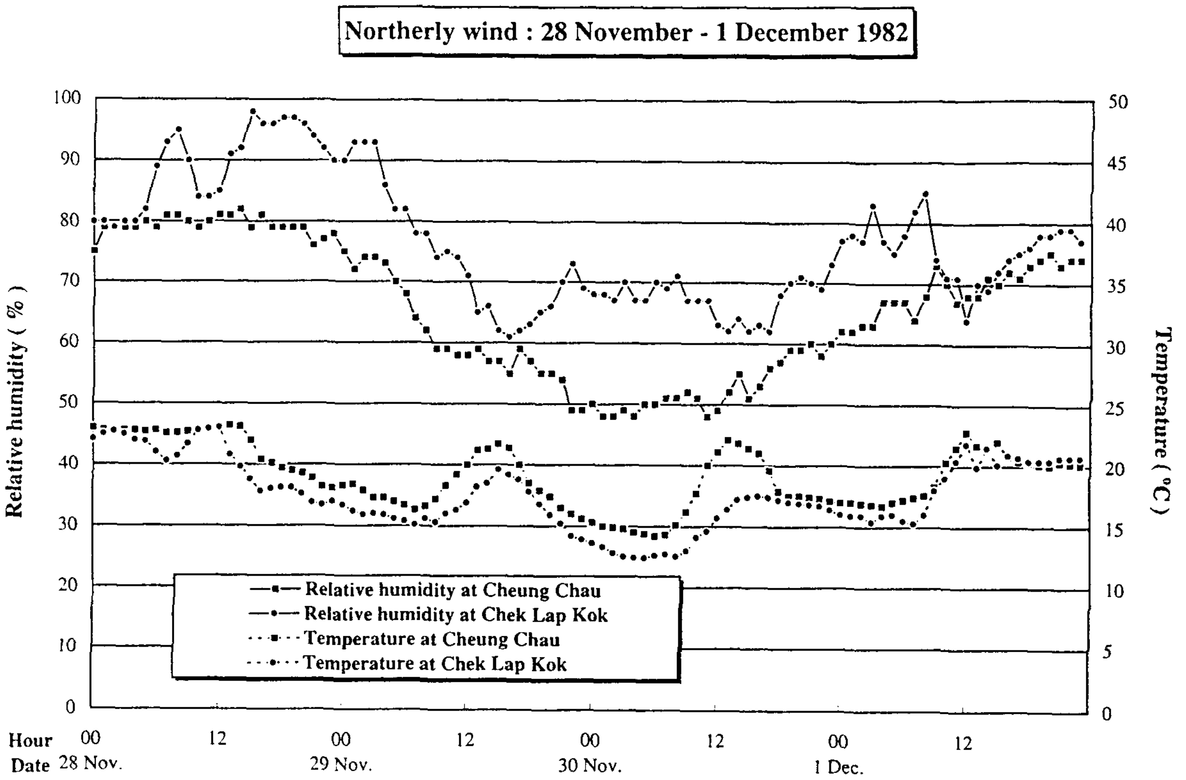


Figure 3 Time series of 12-hourly mean wind speed for Case I.

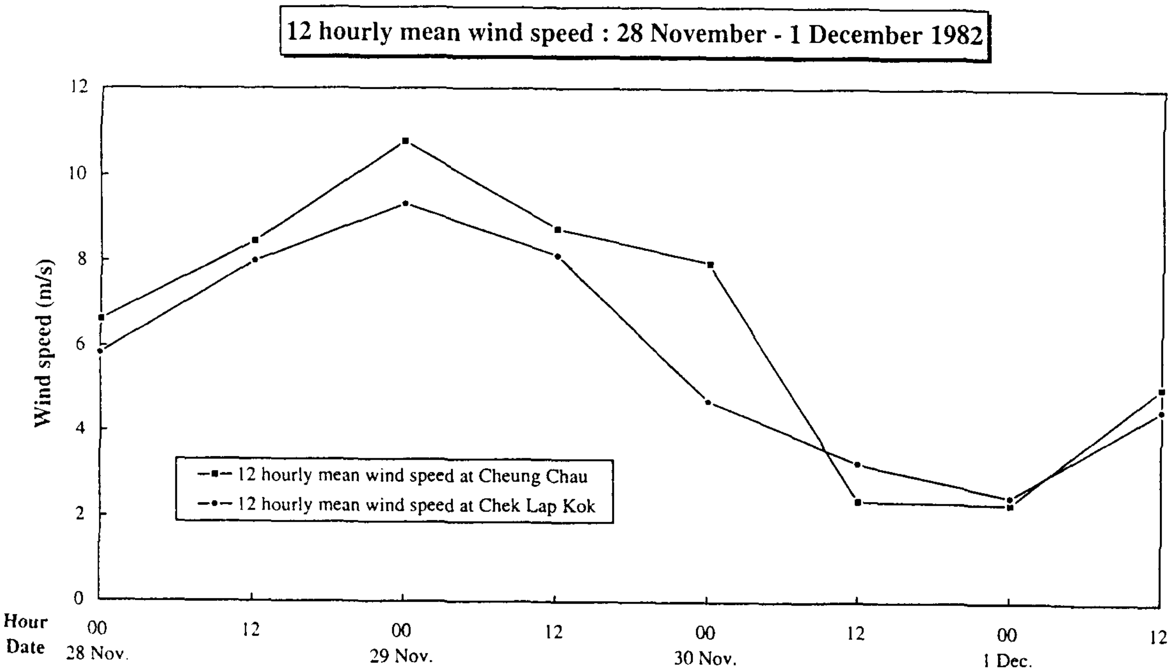
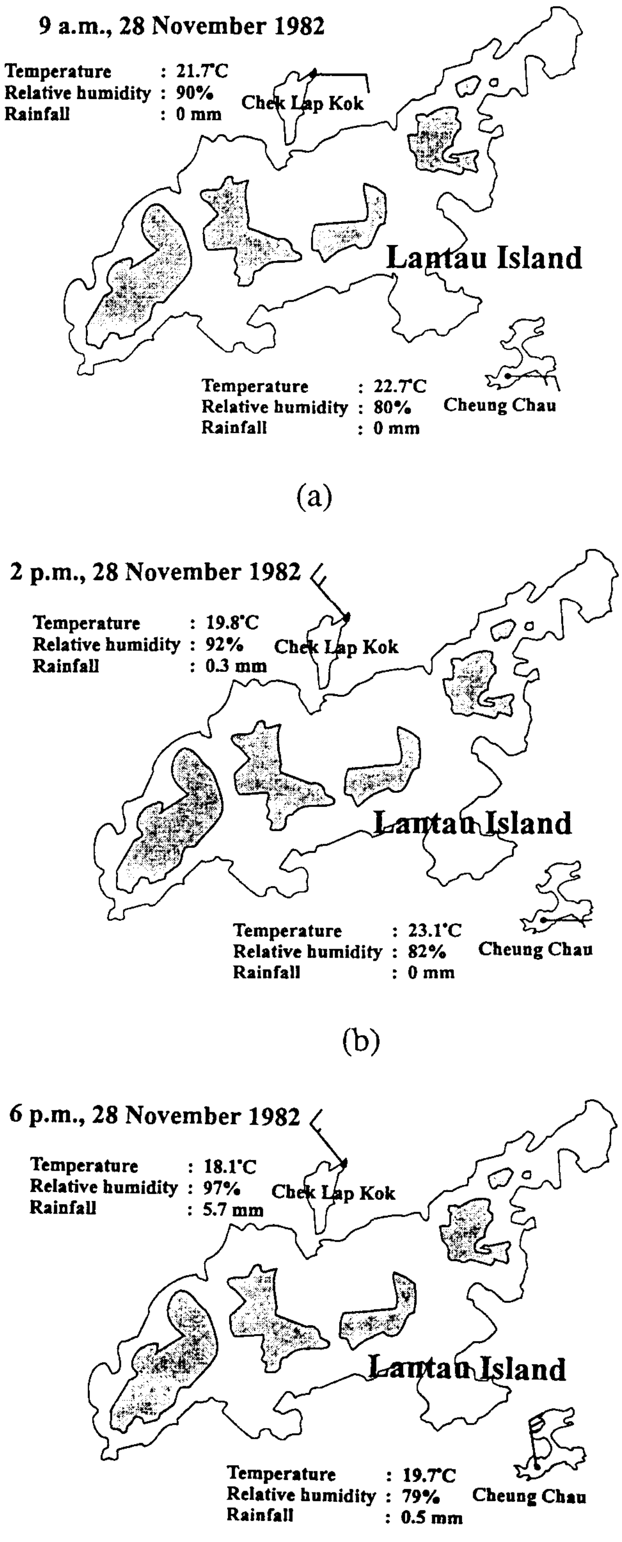


Figure 4 Observations at Chek Lap Kok and Cheung Chau in Case I. Times of observations are given at the top of each figure.



(a)

(b)

(c)

Figure 6 Case II: Time series of temperature and relative humidity at Cheung Chau and Chek Lap Kok from 7 to 14 January 1983.

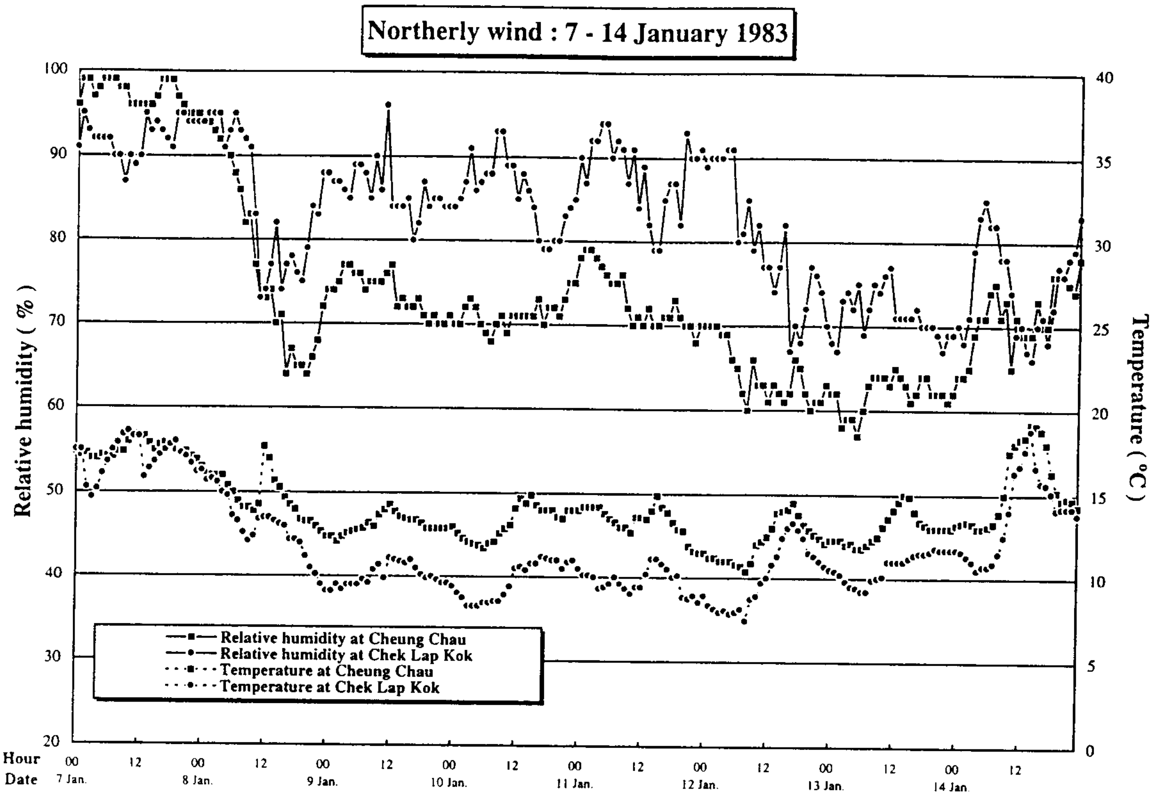


Figure 7 Time series of 12-hourly mean wind speed for Case II.

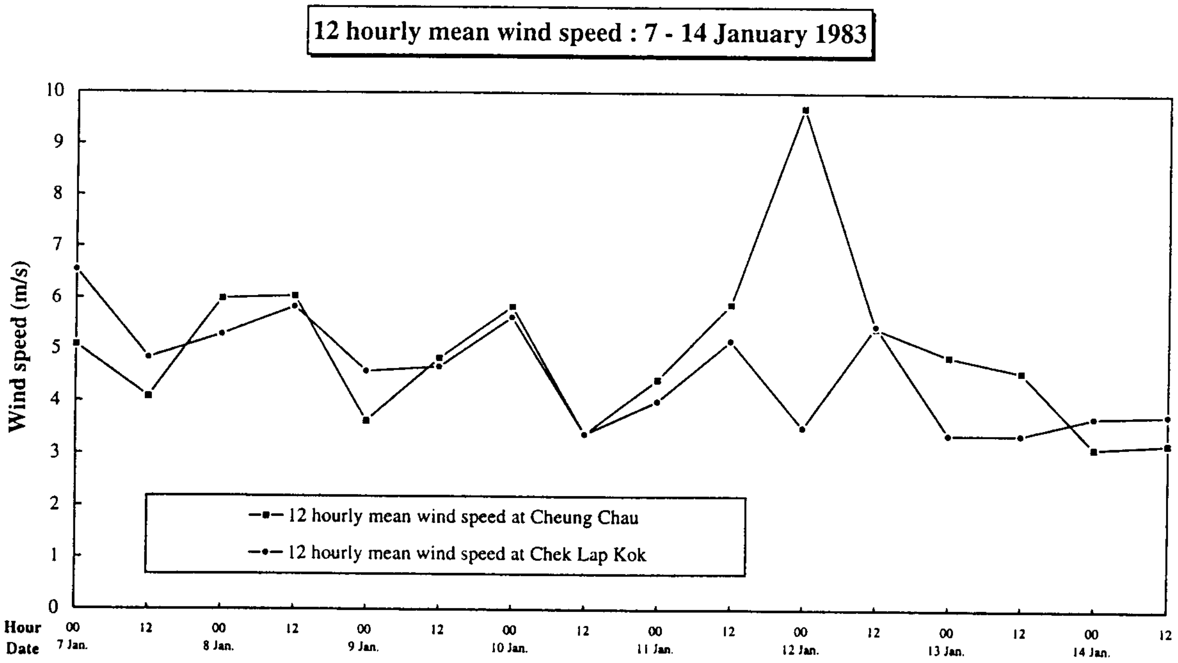
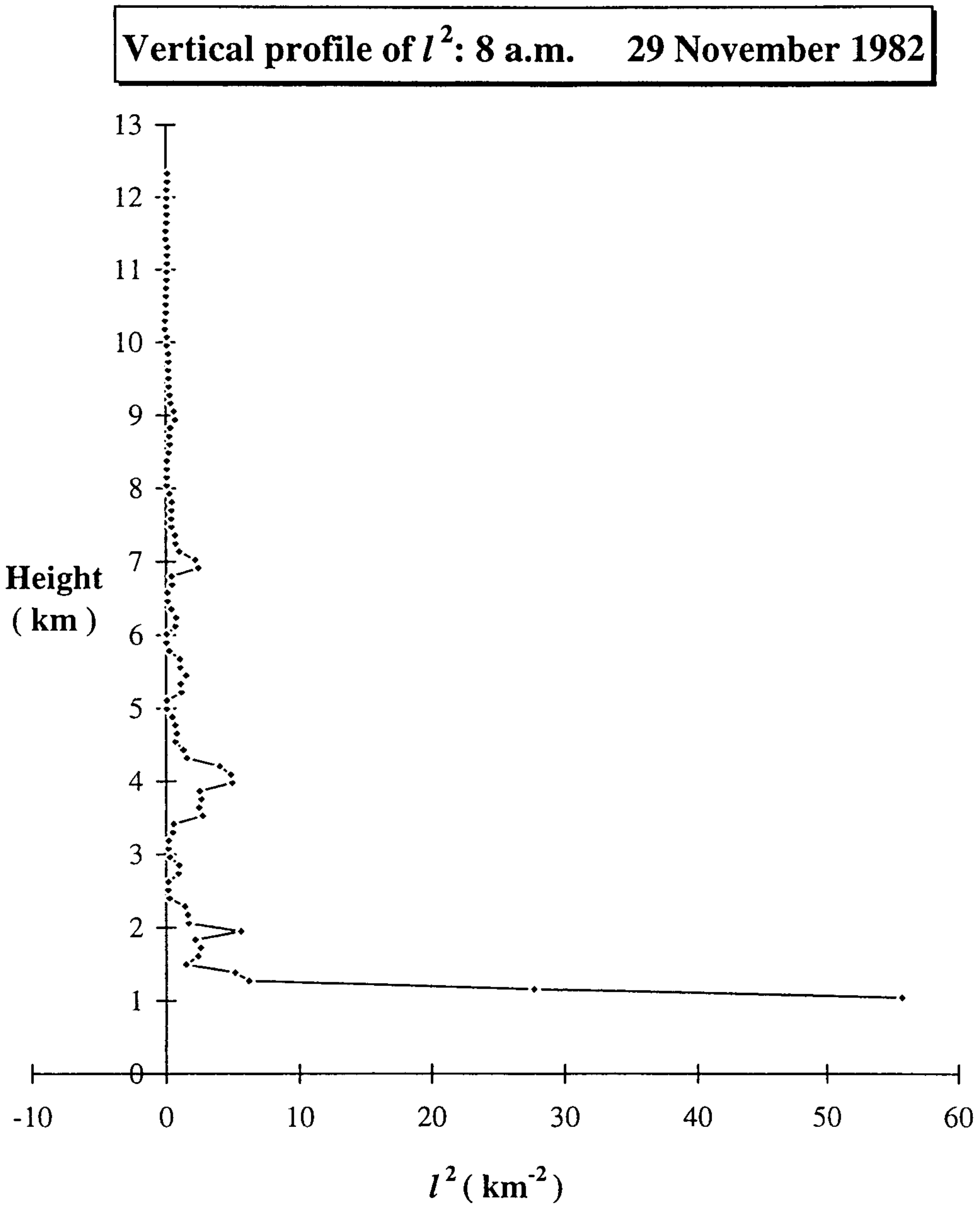


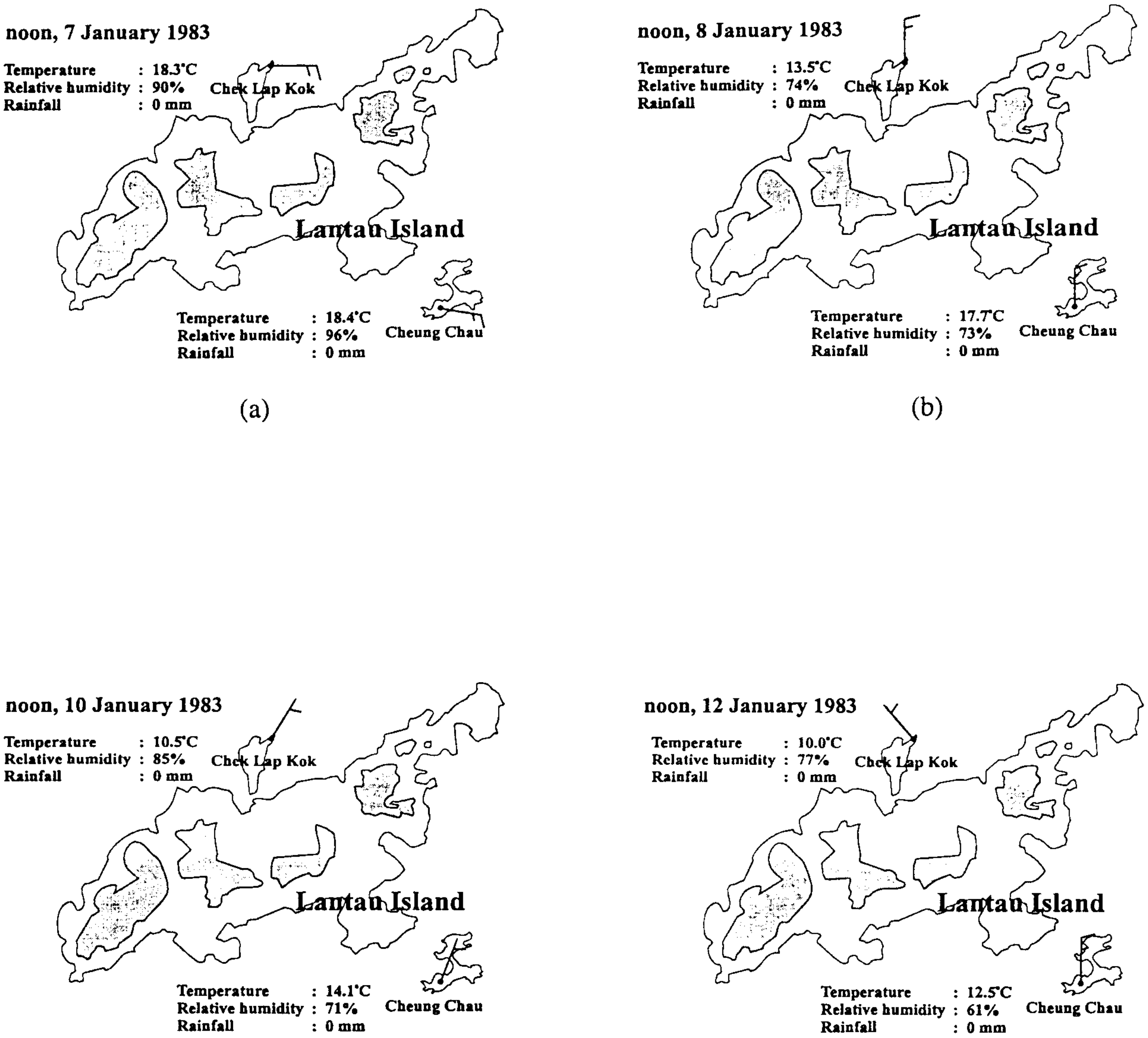
Figure 5 Score parameter profile at 8:00 a.m. on 29 November 1982.



(b) Case II: Northerly winds (7 to 14 January 1983)

As in case I, a Foehn was again observed under a prevailing northerly airstream. Figure 6 shows that temperatures at Cheung Chau were four degrees higher and relative humidity 10 percent lower than at Chek Lap Kok for most of the study period. Winds were generally higher at Cheung Chau particularly at around midnight on 12 January (Figure 7). Selected observations at Chek Lap Kok and Cheung Chau are given in Figure 8.

Figure 8 Observations at Chek Lap Kok and Cheung Chau in Case II.
Times of observations are given at the top of each figure.

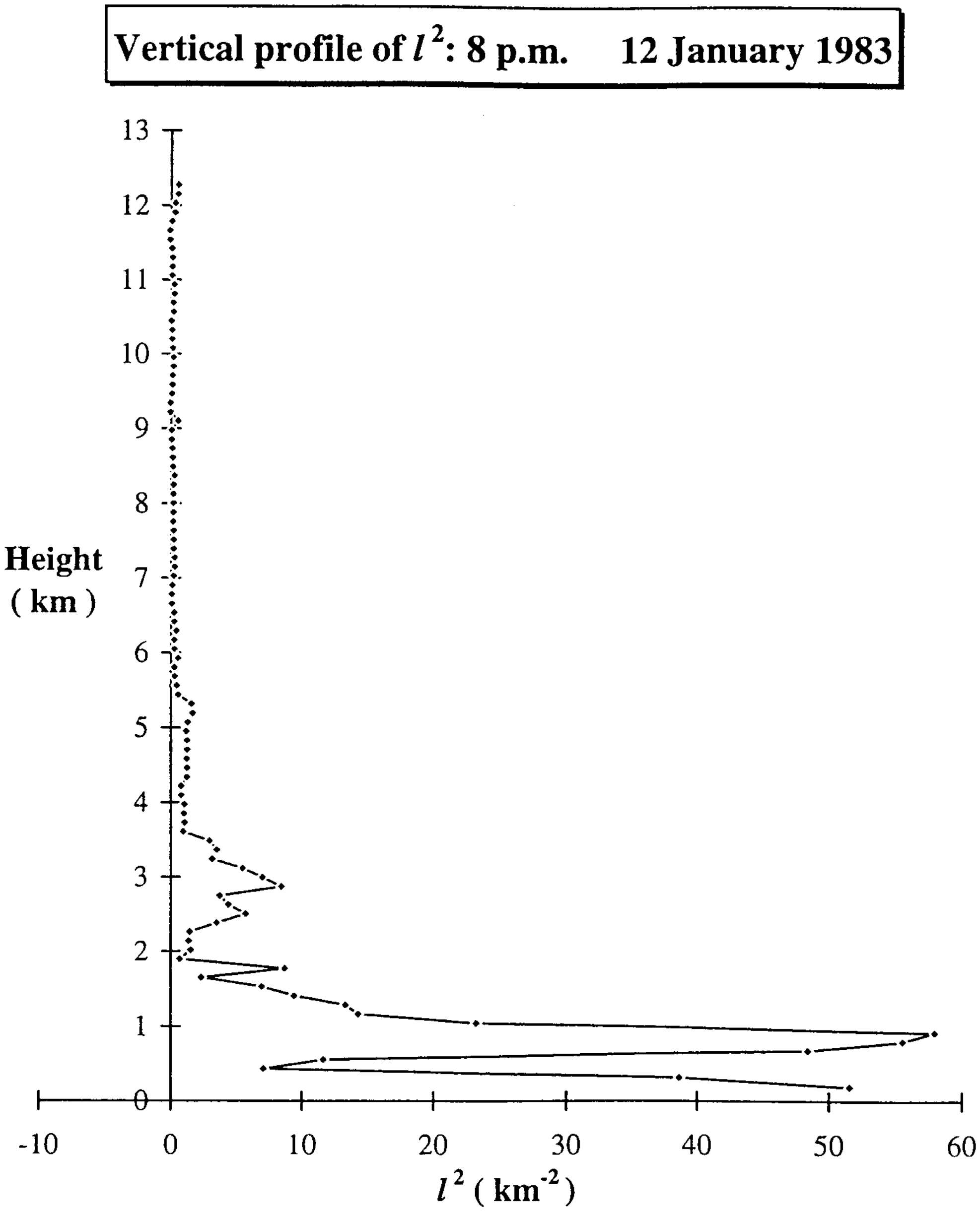


(a) (b)

(c) (d)

Figure 9 shows the Scorer parameter profile at 8 p.m. on 12 January when peak values were noted between 600 and 900 m. The estimated wavelength of the trapped lee waves was around 1.5 km. This wavelength was about one-forth smaller than the half-width of the Lantau ridge, but it was still favourable for the formation of trapped waves.

Figure 9 Score parameter profile at 8:00 p.m. on 12 January 1983.



(c) Case III: Southeasterly wind (17 to 20 February 1995)

The weather station at Chek Lap Kok was closed down in mid-1991due to airport construction and observations commenced at a new station at Sha Lo Wan on north Lantau Island in early 1993.

Figure 10 Case III: Time series of temperature and relative humidity at Sha Lo Wan and Cheung Chau from 17 to 20 February 1995.

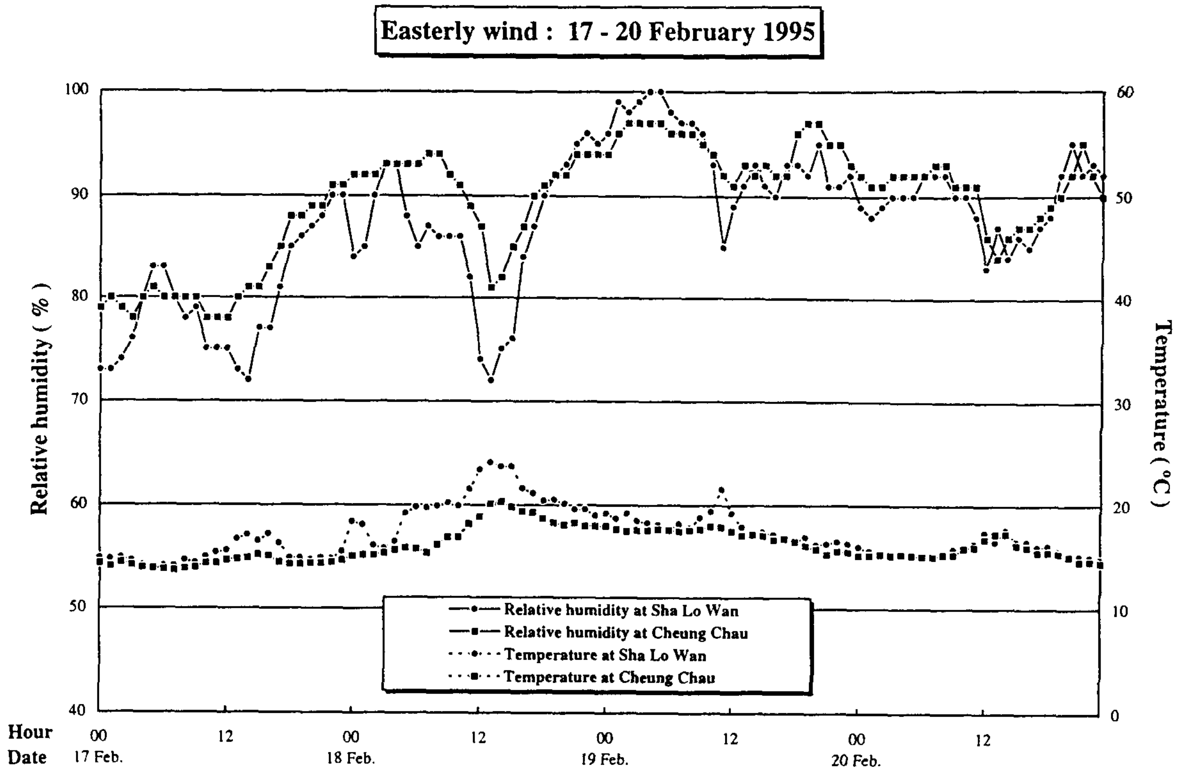


Figure 11 Time series of 12-hourly mean wind speed for Case III.

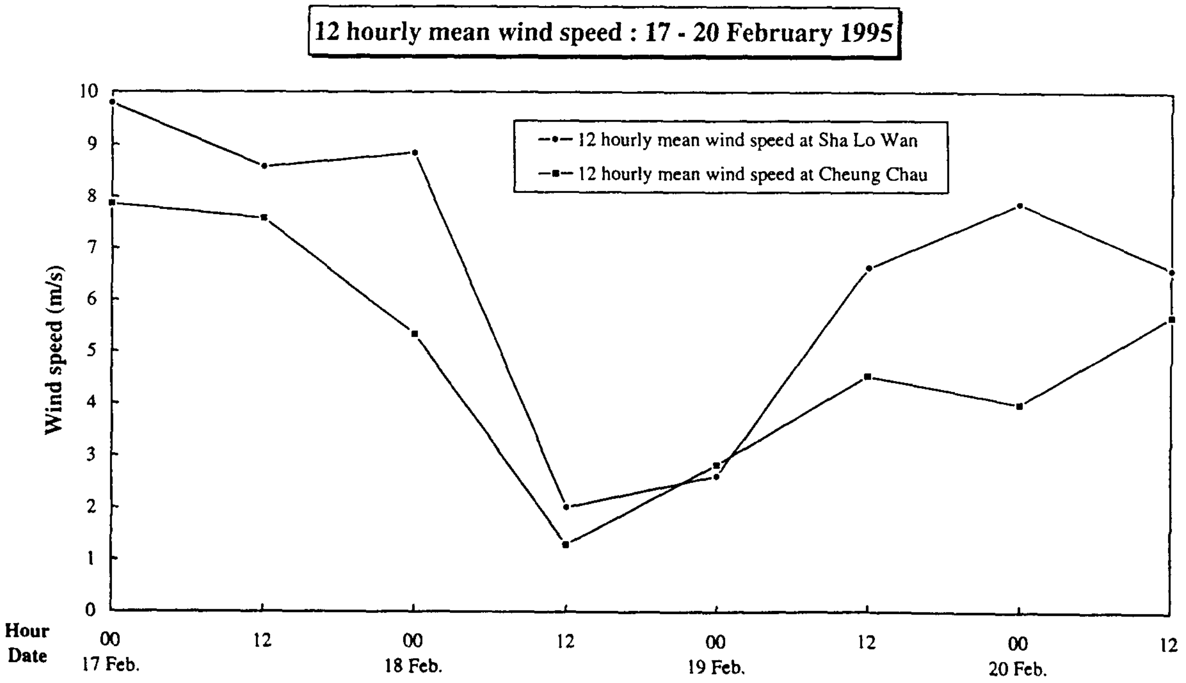
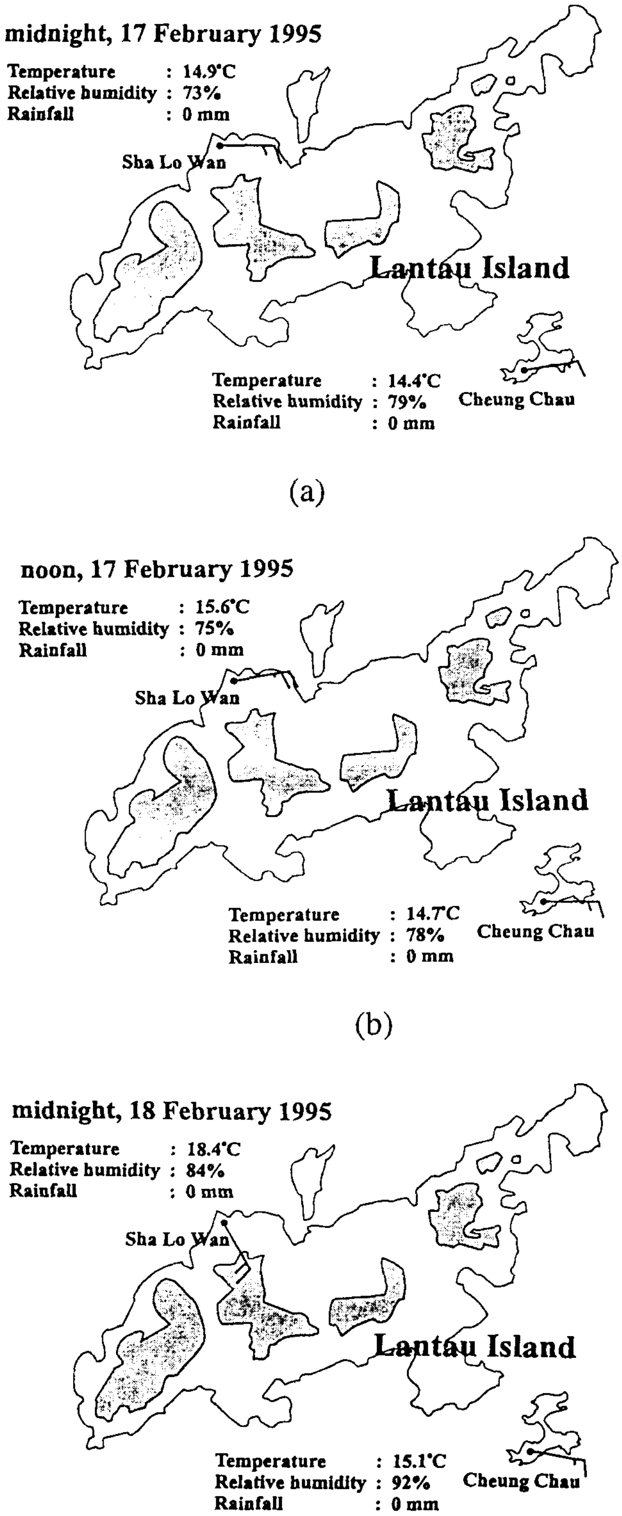
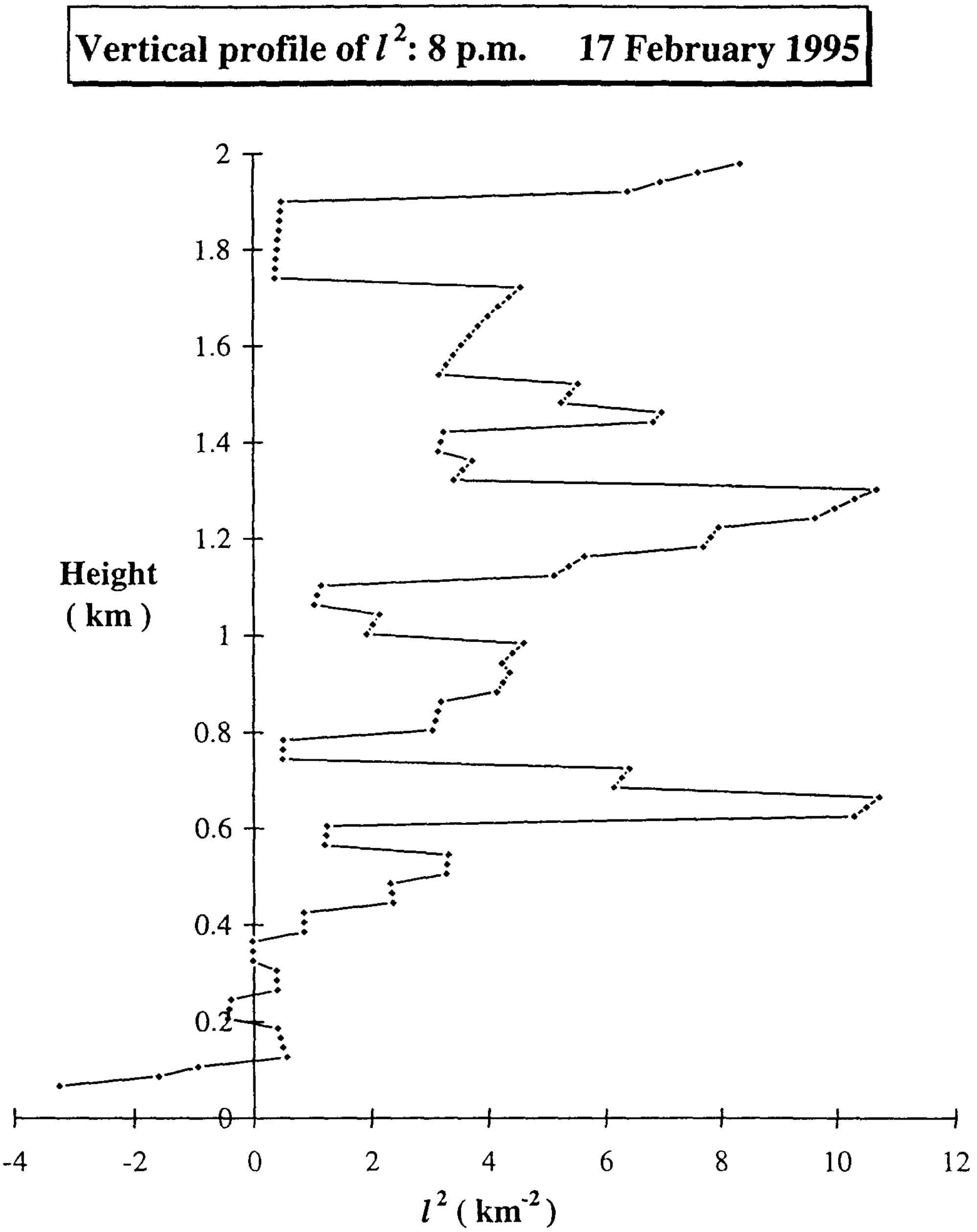


Figure 12 Observations at Cheung Chau and Sha Lo Wan in Case III. Times of observations are given at the top of each figure.



(c)

Figure 13 Score parameter profile at 8:00 p.m. on 17 February 1983.



Unlike the previous two cases, the Foehn effect was observed during this time period under a southeasterly airflow. In this case, Cheung Chau was located upwind of the ridge and hence a turnaround of the physical effects was observed. Figure 10 indeed show s higher temperatures and lower relative humidities at Sha Lo Wan than at Cheung Chau. Wind strength there also was generally higher as shown on Figure 11. The observed effects prevailed for nearly 18 hours from midday of 17 February. Some typical observations at Sha Lo Wan and Cheung Chau on 17 February are given in Figure 12.

Figure 13 shows the Scorer parameter profile at 8:00 p.m. on 17 February. A peak value was found between 600 and 800 m with a corresponding wavelength of about 2.5 km. The wavelength in this example was comparable with the half-width of the ridge.

Conclusions

The Foehn effect in association with trapped lee waves in the airflow over Lantau Island serves to explain the simultaneous increase of temperature and drop in relative humidity on the lee side of the island. This situation occurs when an airstream impinges almost perpendicularly on the ridge barrier under favourable atmospheric conditions. Although the wind direction downstream of the barrier remains similar to the prevailing airflow, a slight enhancement of the wind speed is observed due to shooting flow.

In the cases studied, the Foehn phenomenon was depicted by *in-situ* ground measurements on either side of Lantau Island and the appearance of trapped waves supported by deductions from mathematical models. The computed Scorer parameter showed maximum values at around the ridge level. The derived lee wavelengths were very similar to the half-width of the ridge in most cases.

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